京都大学宇宙総合学研究ユニットセミナー
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乱流とMHDモードの
非線形相互作用とダイナミックス

突発性・爆発現象の理解に向けて

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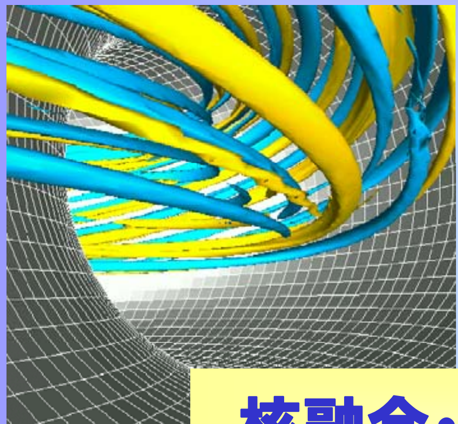
プラズマ・核融合基礎学分野

K-4



岸本・李研究室

Kishimoto-Li
Labratory

核融合・光量子・宇宙の理解をめざした理論プラズマ物理学の探求

http://www.center.iae.kyoto-u.ac.jp/kishi/

内　容

- ▶ はじめに
  - ・トカマクにおける“反転磁気シア”磁場と安定性
  - ・揺らぎ（乱流）と2次構造（帯状流・帯状磁場・KHモード…）の役割
- ▶ ダブルティアリングモード（DTM）の構造と非線形発展
  - ・ 構造駆動の非線形発展と速い磁気再結合（電流点形成）（2002 PRL）
  - ・ 電流シートを維持した早い磁気再結合（Wang et al. 2007 PRL ）
- ▶ MHDモードと乱流との相互作用と素過程
  - ・ 渦流と乱流との相互作用
  - ・ ティアリングモードと温度勾配モード乱流の相互作用とエネルギー輸送経路（構造とエネルギー）
- ▶ まとめ

平成１６年３月３日（水）　：　新橋・航空会館
３月４日（木）　：　日本原子力研究所計算科学技術推進センター

多階層・複合系プラズマの切り開く学際領域の開拓

非線形不安定性と爆発（突発）現象の理解

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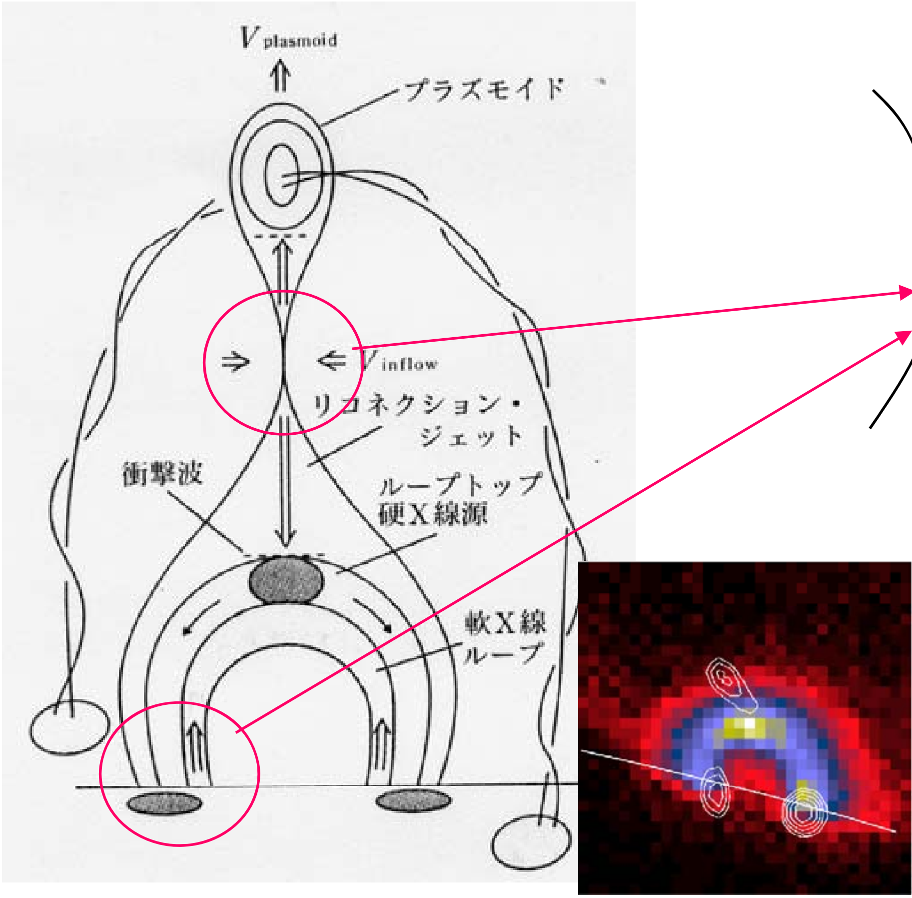
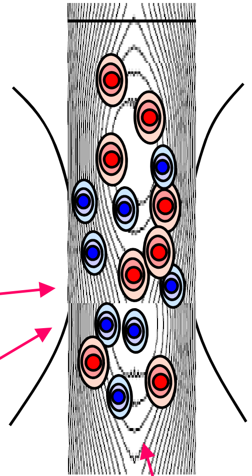
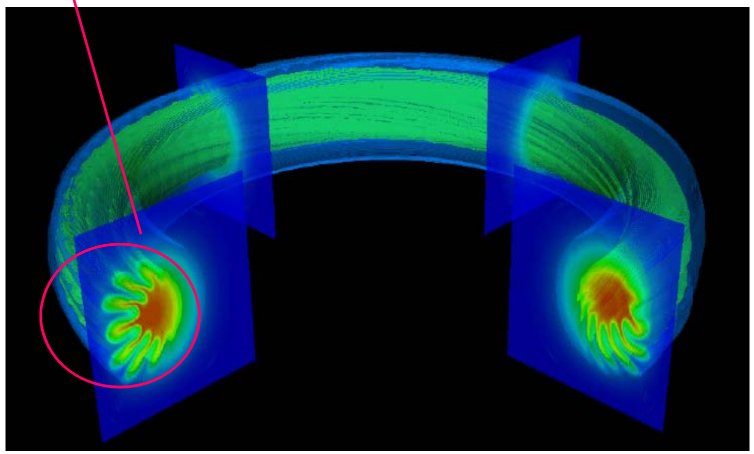
宇宙・天体プラズマと磁場閉じ込めプラズマ



太陽フレアモデル（柴田等）



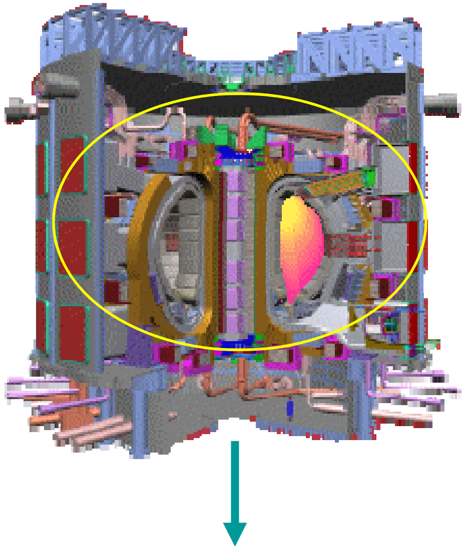
マクロなMHDとミクロスケール
の乱流の相互作用

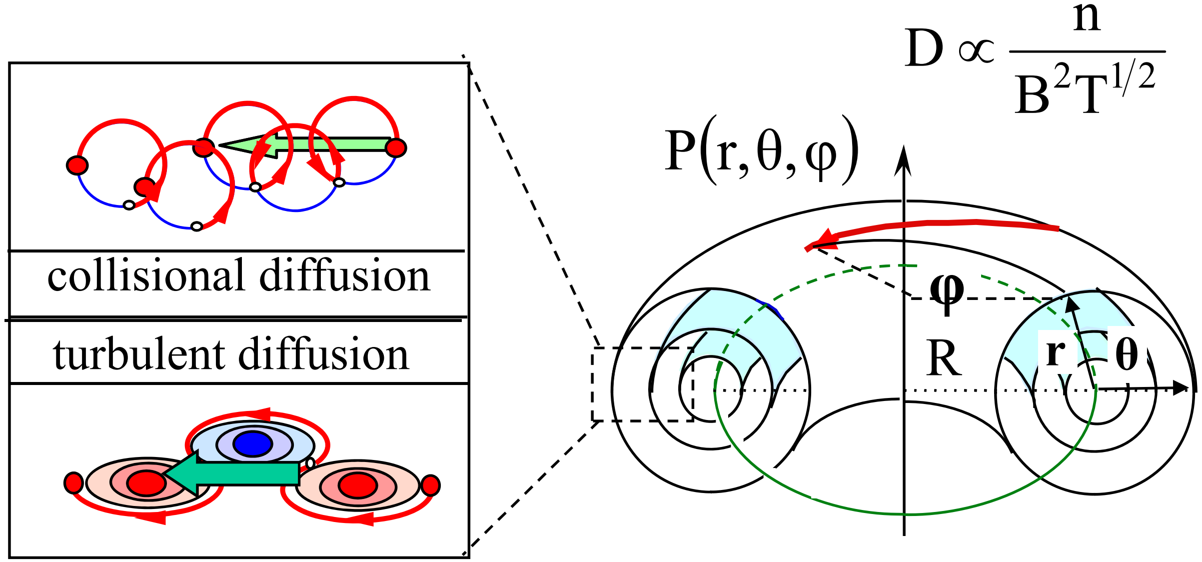


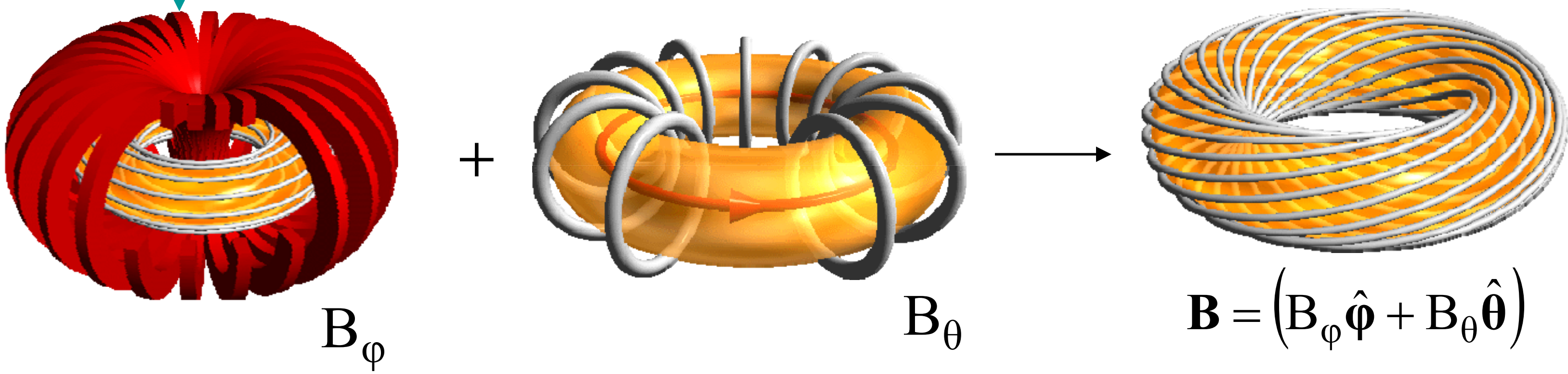
[*Kagei (JAEA)-Kishimoto*]

Fusion device(Tokamak) & magnetic fields

Magnetic fields are designed to minimize
various plasma fluctuations based on linear theory



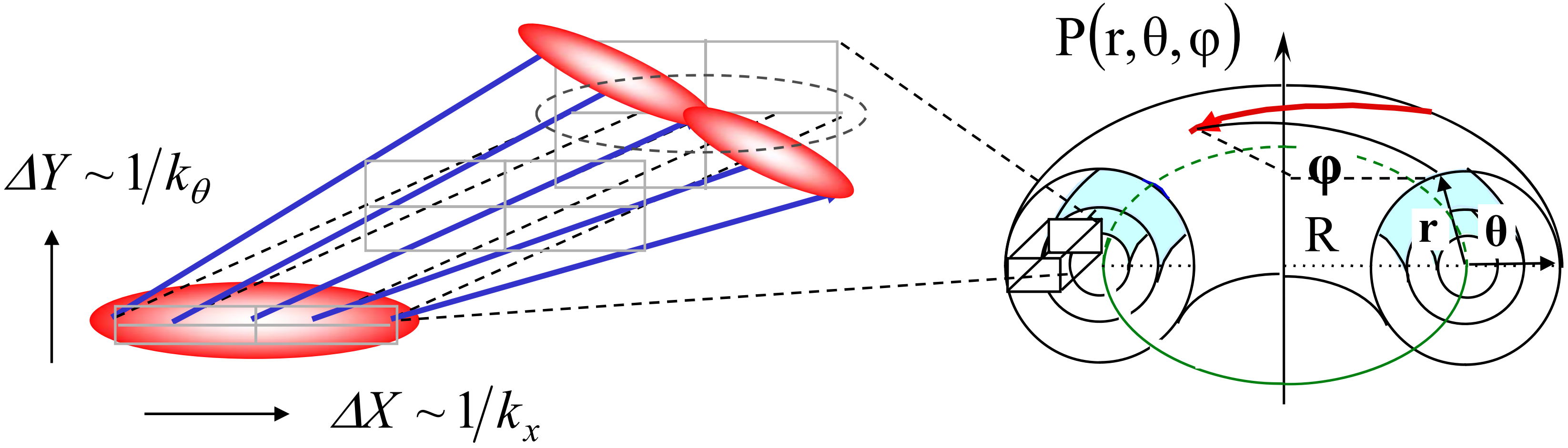




Toroidal magnetic field | Poloidal magnetic field | Tilted Magnetic cage

Fusion device(Tokamak) & magnetic fields (2)

Magnetic fields are designed to minimize
various plasma fluctuations based on linear theory



$$q(r) \equiv \frac{r}{R}\frac{B_T}{B_P} \sim \frac{m}{m}$$

磁場の傾斜
（安全係数）

$$\hat{s} = \frac{r}{q}\frac{\partial q}{\partial r}$$

傾斜率の変化
（磁気シア）

$$\Delta X \cong \frac{1}{\hat{s}k_\theta} \propto \frac{\rho_j}{\hat{s}}$$

揺らぎのサイズ

$$D \cong \frac{\Delta X^2}{\tau_c}$$

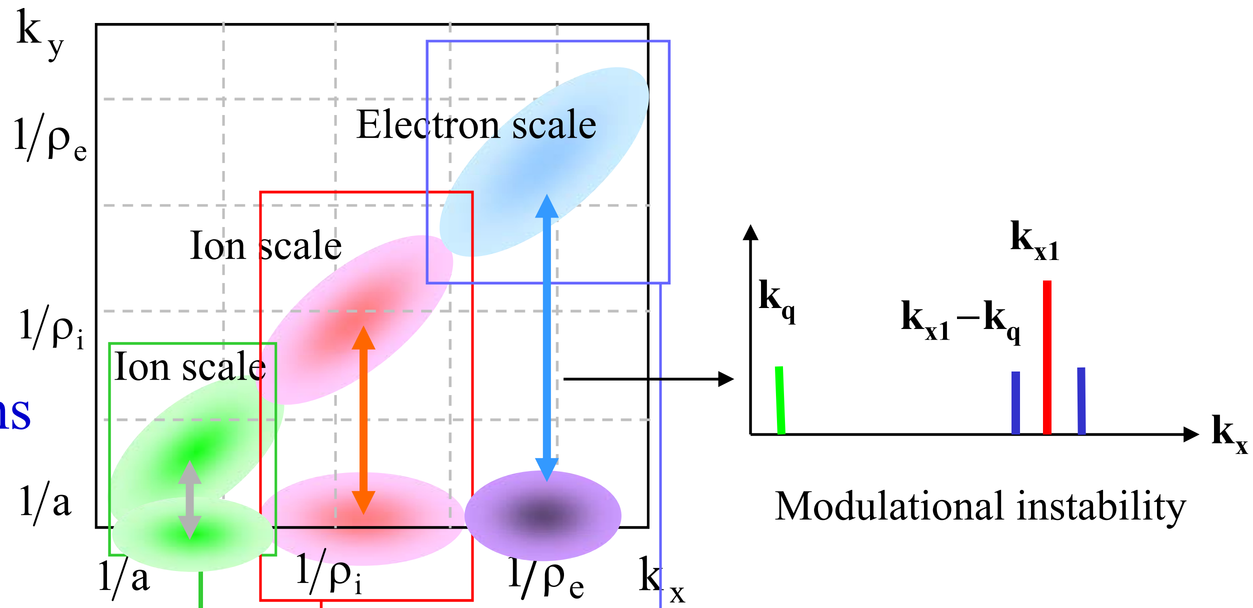
拡散係数

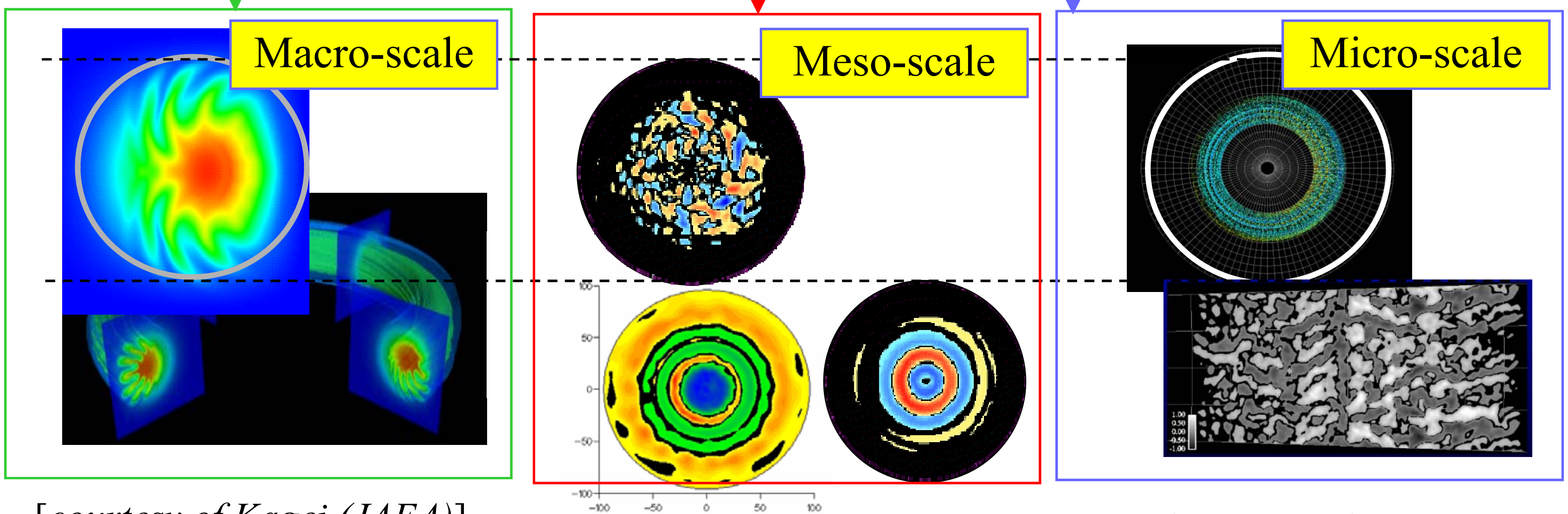
Various fluctuation with different scales

- ► many eigen modes,

  $\Delta k \sim 10^5 \quad \Delta\omega \sim 10^8$

- ► coexistence of multi-scale fluctuations



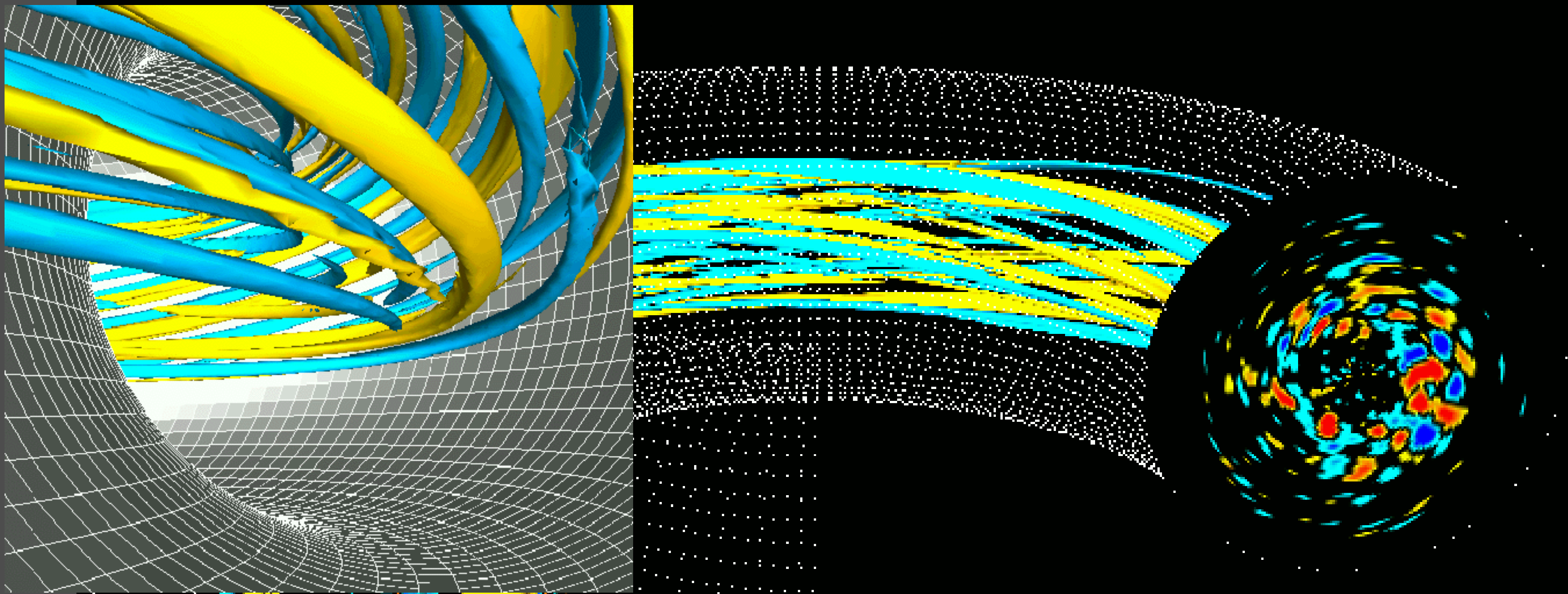


[*courtesy of Kagei (JAEA)*]

[Y. Kishimoto et al.'96, PoP]

[Y. Idomura et al.'04, IAEA, NF]

Global turbulent simulation of ion temperature gradient mode (ITG)



ΔX : correlation length

τ_c : correlation time

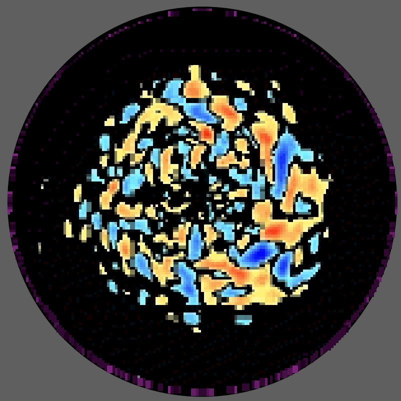
$$D \cong \frac{\Delta X^2}{\tau_c}$$

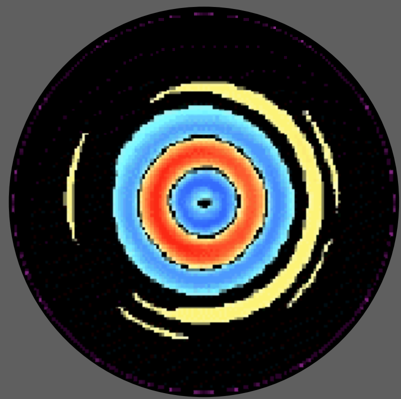
$$\Delta X \cong \rho_i^{1-\alpha} L_T^{\alpha} \cong \sqrt{\rho_i L_T}$$

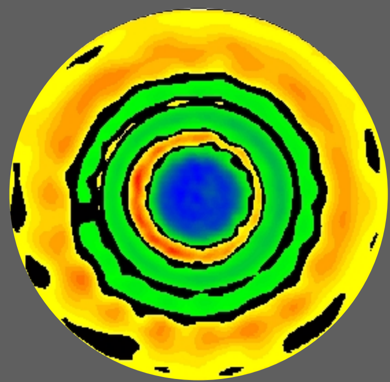
$$\rho_i < \Delta X < a, L_T$$

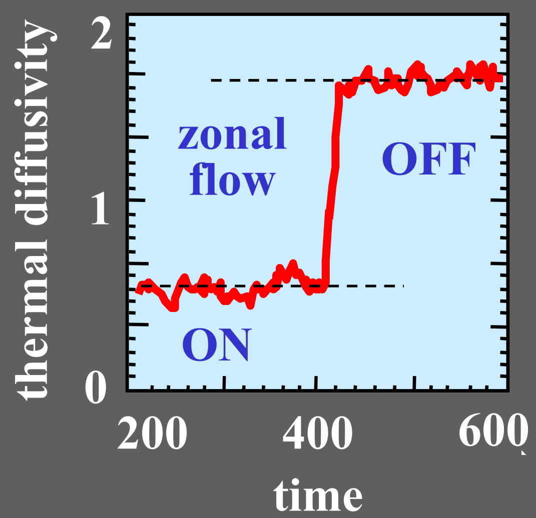
$$D \propto \left(\frac{\rho_s}{a}\right)^{1-\alpha} \frac{T}{B}$$

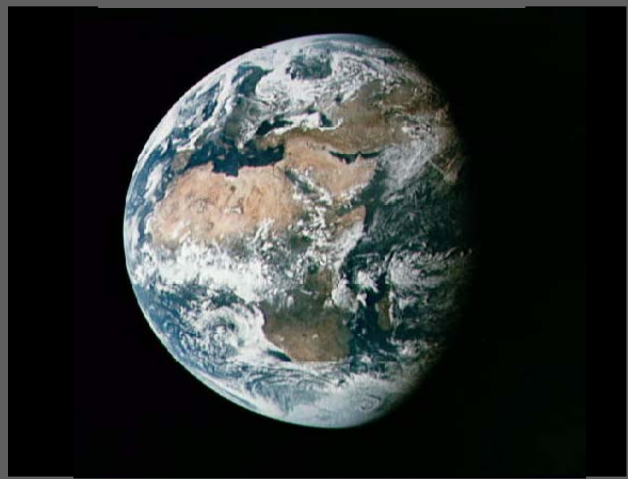
Global flow generation from turbulence









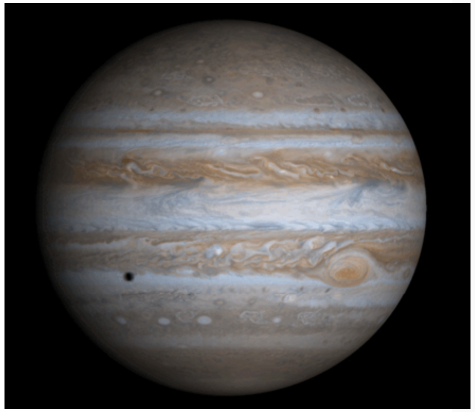


World dominated by
turbulence and flows

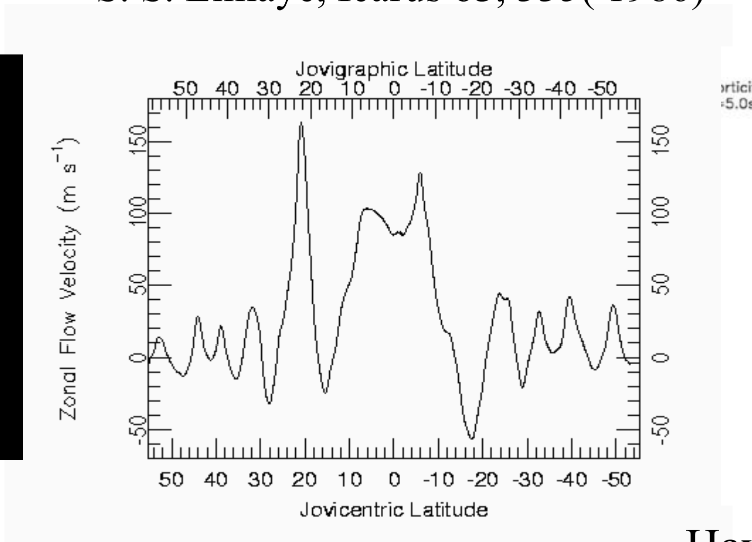
Zonal flows show a complex radial structure

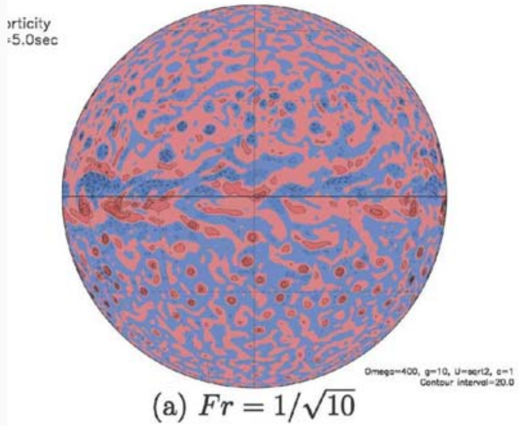
- Radial structure of zonal flows are very complex !

S. S. Limaye, Icarus **65**, 335(1986)



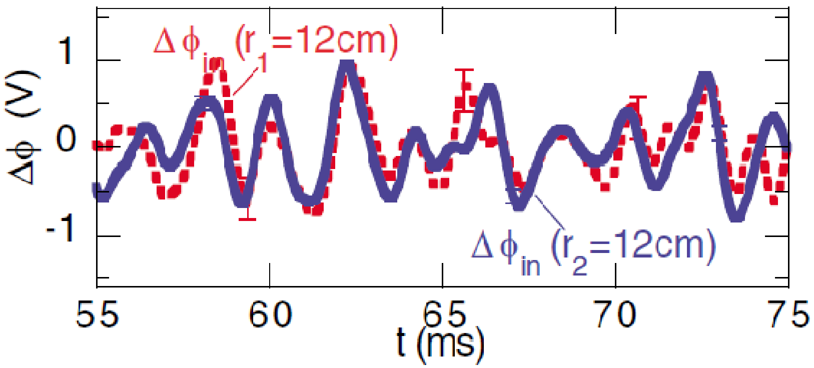
(NASA)





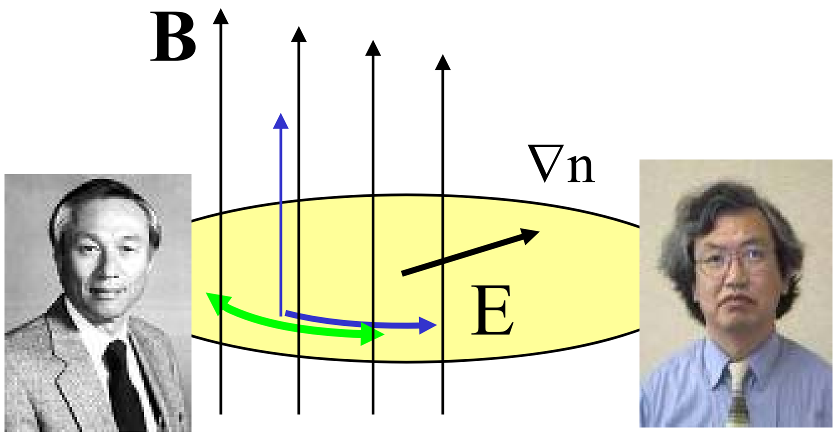
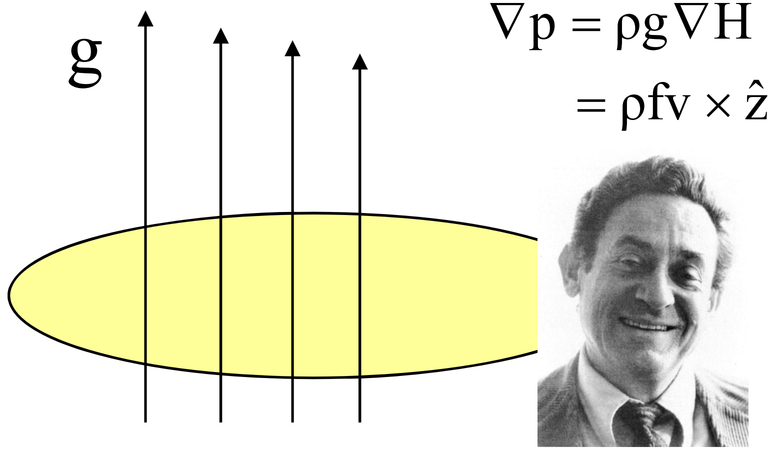
Hayashi, Nishizawa, Takehiro, Yamada, Ishioka, and Yoden (2007), *J. Atmos.Sci.*, 64, 4246-4269.

[*courtesy of Kagei (Kyoto)*]



A.Fujiwara,
Phys. Rev. Lett. **93** 165002, 2004

Turbulence in quasi-two dimensional bounded system

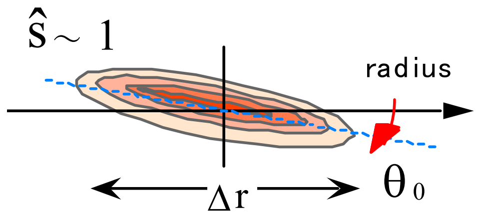
Drift wave : Hasegawa-Mima eq.	**Rossby wave : Charney equation**
$(1-\nabla^2)\frac{\partial \varphi}{\partial t} + v_d \frac{\partial \varphi}{\partial y} - [(b \times \nabla \varphi)\cdot \nabla]\nabla^2 \varphi = 0$	$(1-\nabla^2)\frac{\partial h}{\partial t} + v_R \frac{\partial h}{\partial y} - [(\hat{z} \times \nabla h)\cdot \nabla]\nabla^2 h = 0$
	 $\nabla p = \rho g \nabla H = \rho f v \times \hat{z}$
$v_E = \frac{c}{B}\hat{z} \times \nabla \phi \qquad v_p = -\frac{c}{\omega_{ci} B}\frac{d}{dt}\nabla_\perp \phi$	$v_c = \frac{g}{f}\hat{z} \times \nabla H \qquad v_p = -\frac{g H_0}{f^2}\frac{d}{dt}\nabla h$

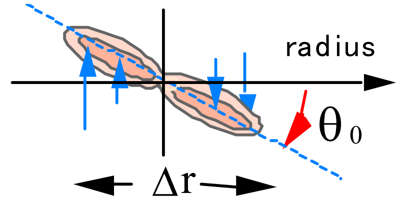
Conserving quantities

Energy : $W = (\nabla \phi)^2 + \phi^2$ Enstrophy : $U = (\nabla \phi)^2 + (\nabla^2 \phi)^2$

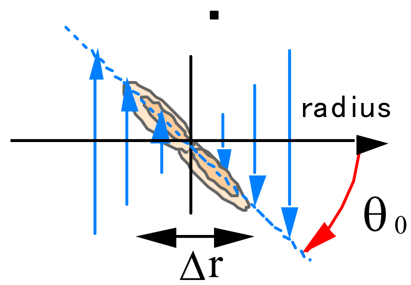
Interaction between flows & fluctuations

Shearing decorrelation of turbulence



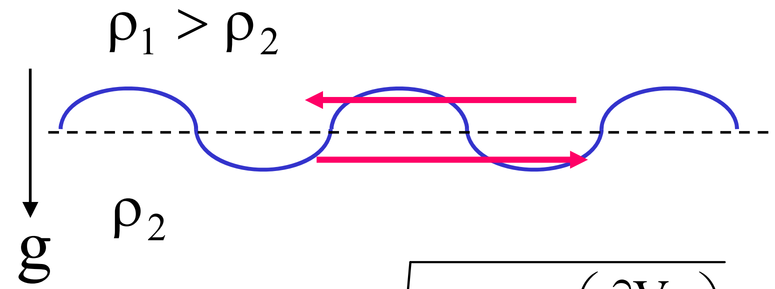


$$\left(1+k_\perp^2\right)\frac{\partial}{\partial t}\phi_{k_x} = -ik_y\phi_{k_x} + \upsilon'_{sf}\left(1+k_\perp^2\right)k_y\frac{d\phi_{k_x}}{dk_x} + \left[\phi,\nabla_\perp^2\phi\right]_{k_x}$$



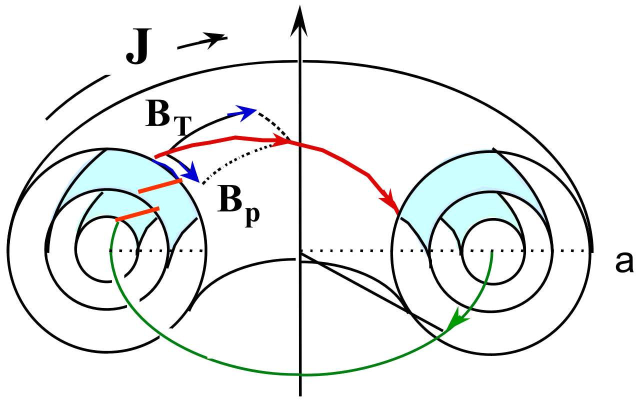
Shearing decorrelation is a local interaction in k space

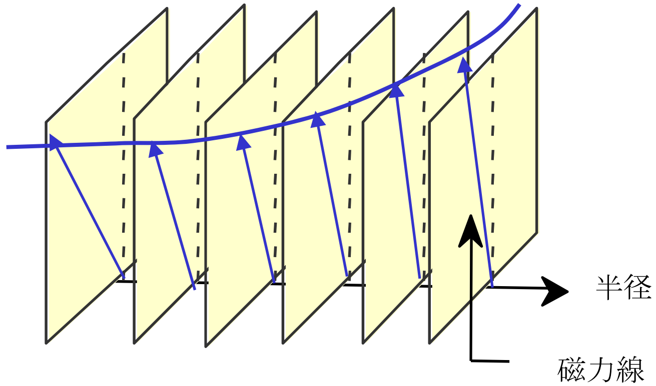
$$\left\langle \delta k^2 \right\rangle = t^2 k^2 V_E'^2$$



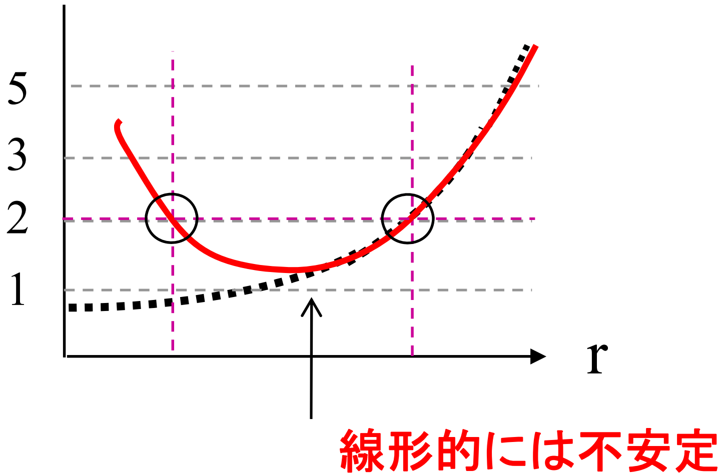
$$\gamma = \sqrt{k_y g - \alpha\left(\frac{\partial V_y}{\partial x}\right)}$$

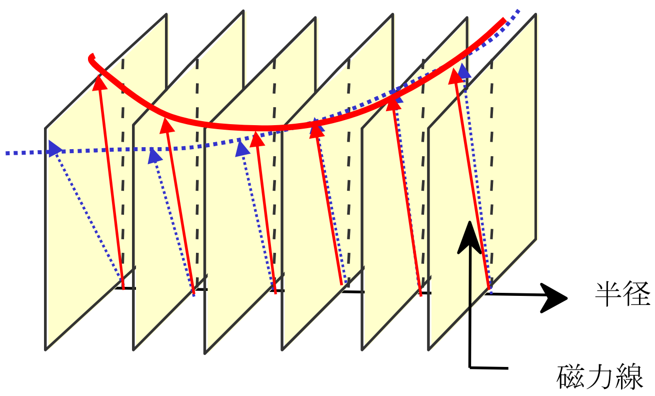
Configuration of "Magnetic Shear Reversal"





$$q(r) \equiv \frac{r}{R}\frac{B_T}{B_P} \sim \frac{m}{n} \qquad \hat{s} = \frac{r}{q}\frac{\partial q}{\partial r}$$

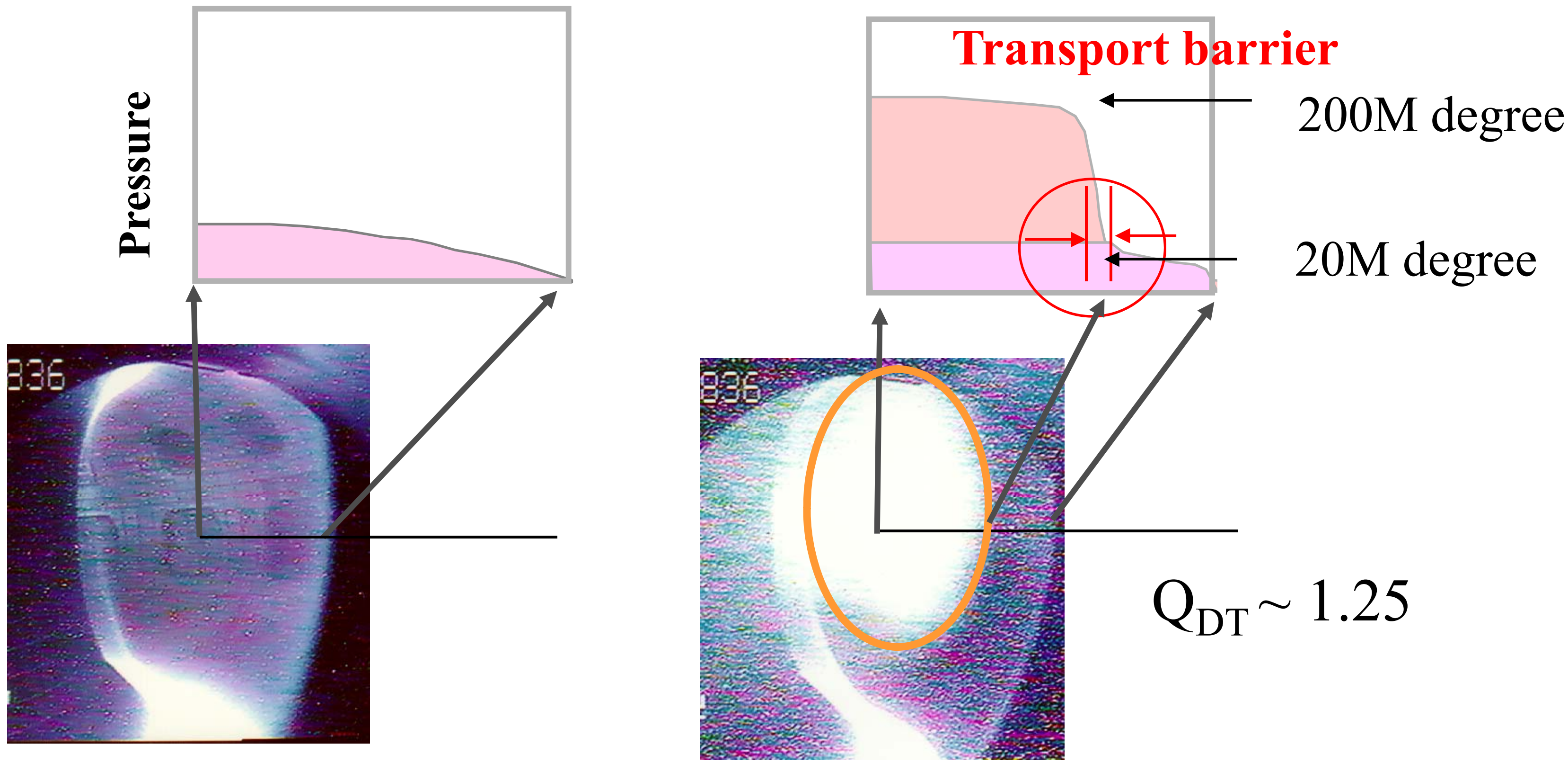




- Tow resonant surface
- Zero magnetic shear region

A high performance plasma is realized by having "structure"

JT-60 High Performance Shot

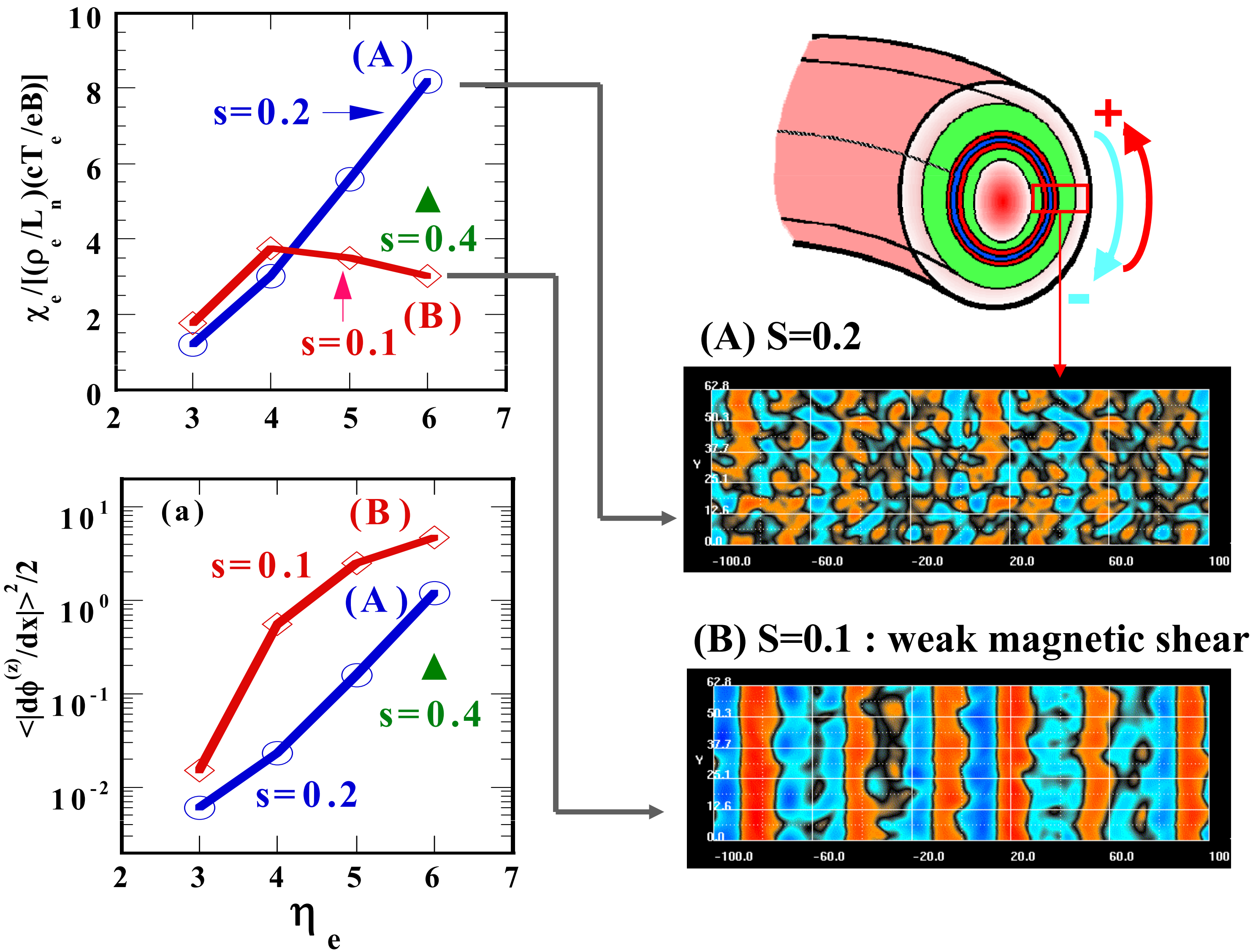


Low confinement plasma

High confinement plasma

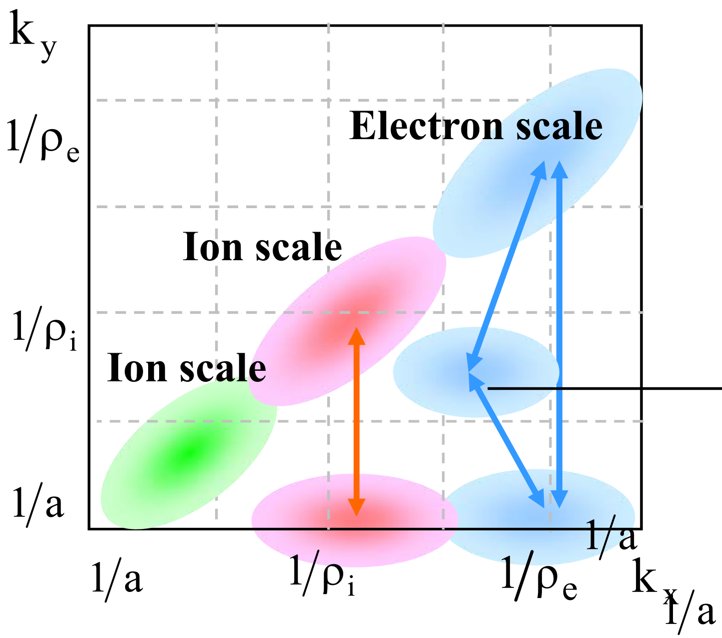
電子系の揺らぎの構造と制御

[*Kishimoto,Li, et al., IAEA ’02*]

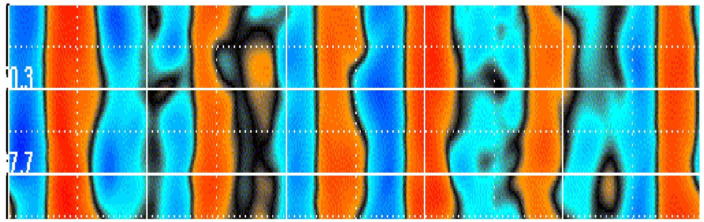


Turbulence dominated by large scale structure

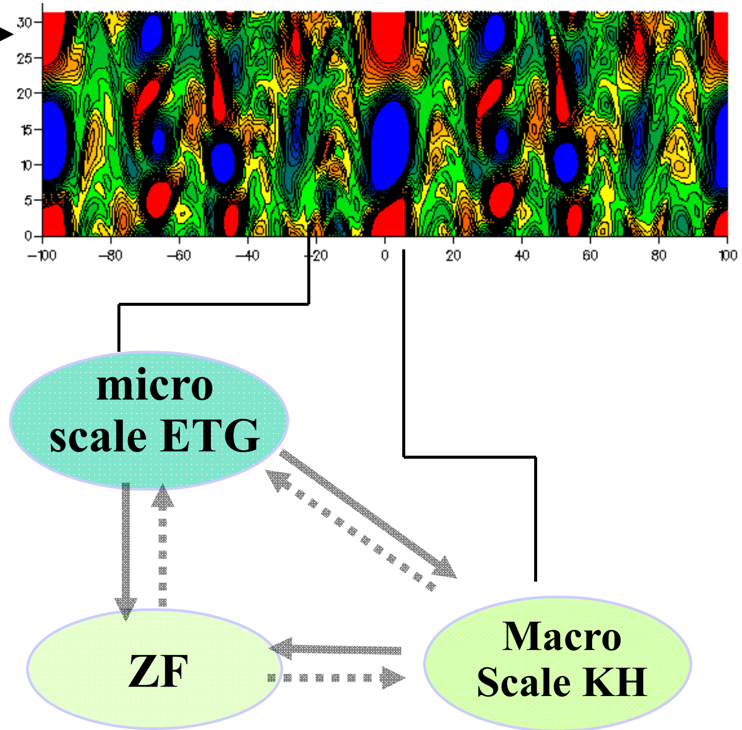
$\hat{s} = 0.1 \quad \eta_e = 6$



turbulence + zonal flow



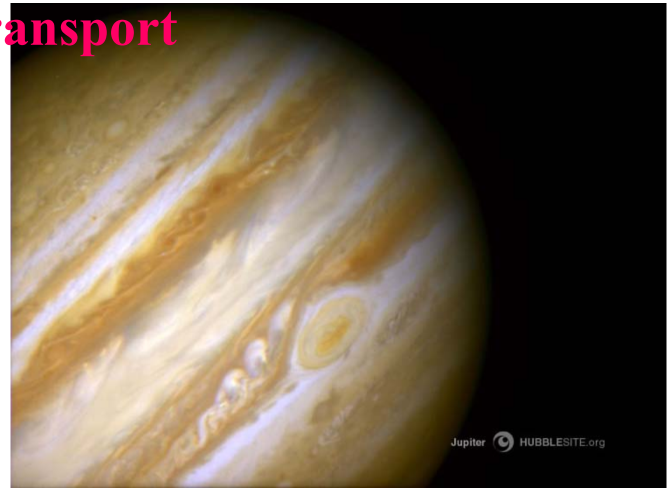
turbulence



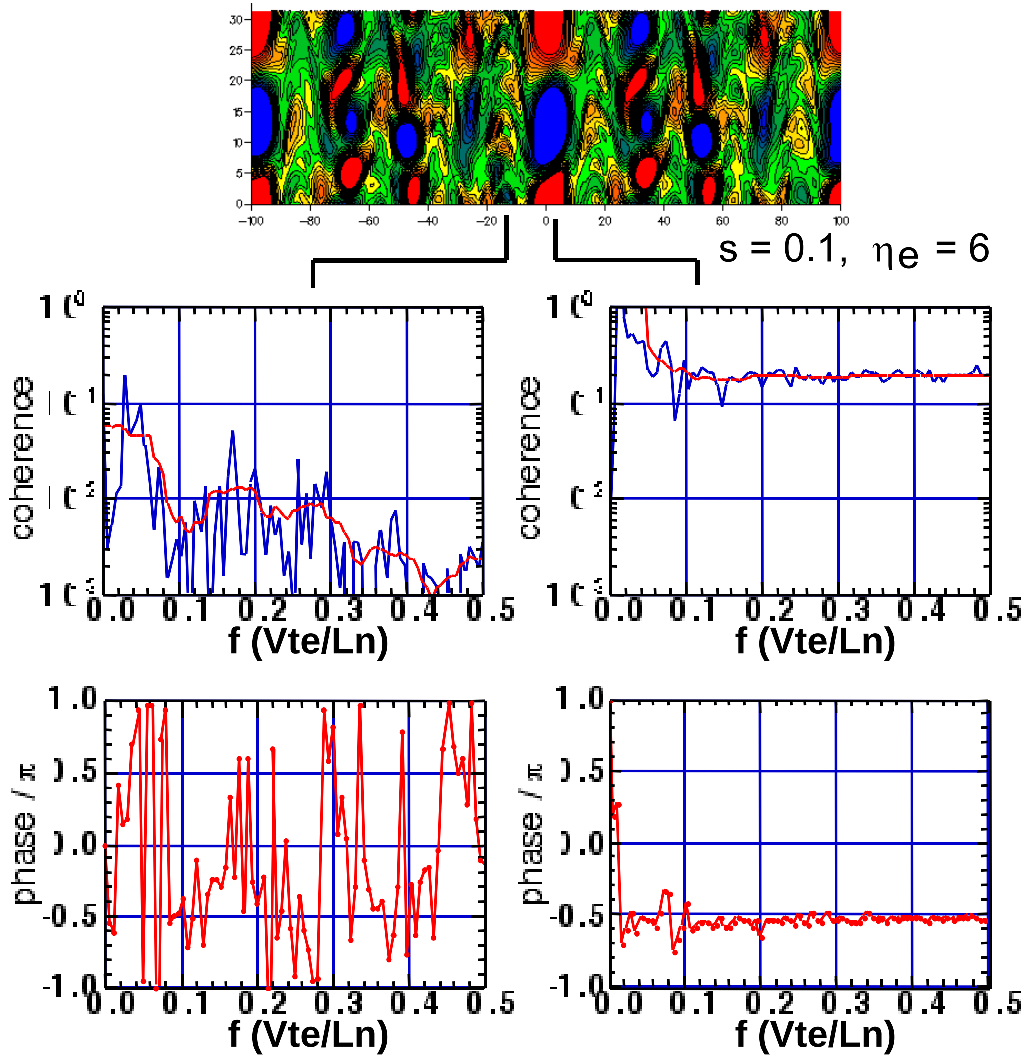
- **Emergence of large scale vortices**
- **Mixed turbulence with**
  - **micro-scale ETG**
  - **ETG driven ZF**
  - **ZF driven Large scale structure**

Coherency and phase between E_y and n

High coherency, but keeping phase relation that produces no transport

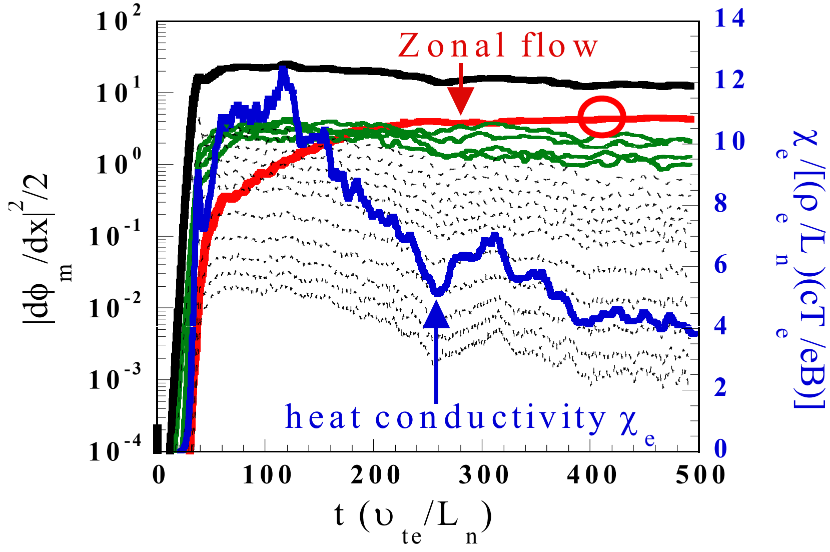


Zonal flow dominated plasma

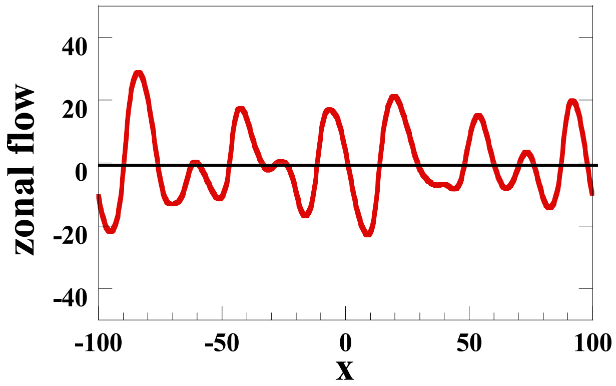


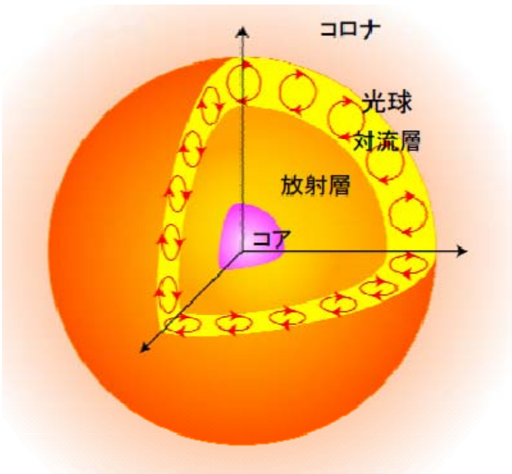
Micro-scale region **Macro-scale region**

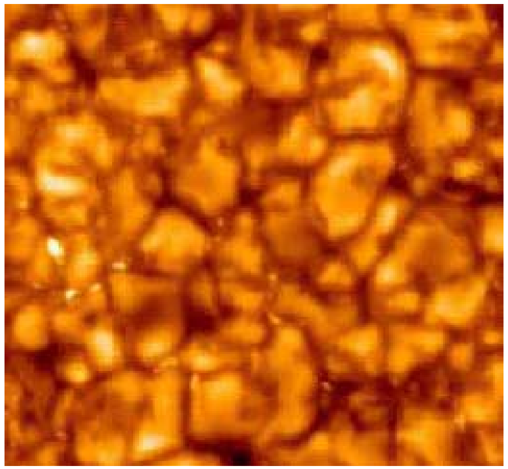
“Marginal nature” of global mode

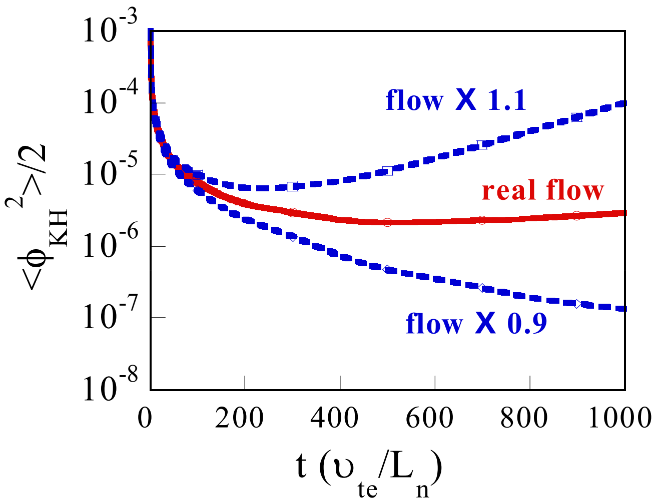


- **Linear analysis intoroducing ETG-driven zonal flow pattern**









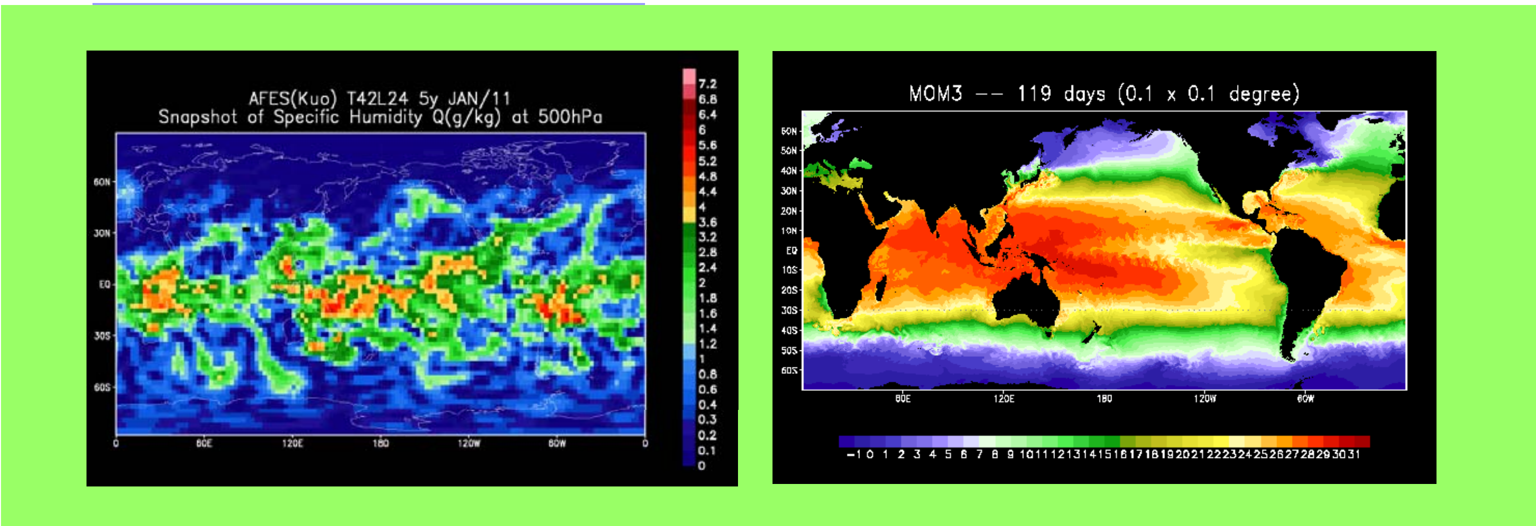
Marginally unstable KH

高圧力状態における自己組織化現象

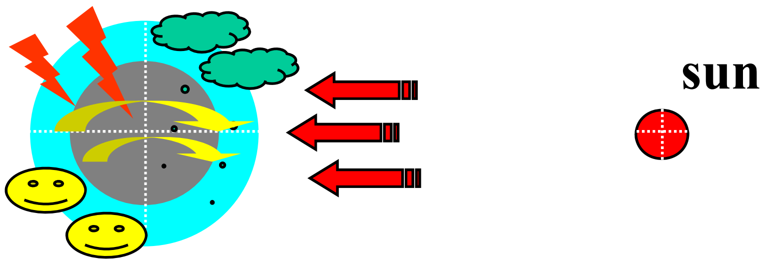
Total fluctuation

= turbulent fluctuation + zonal fluctuation
(→ induce transport)

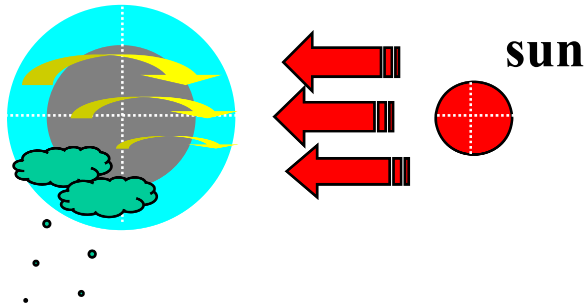
$$\eta_{zF} \equiv \frac{E^{(ZF)}}{E^{(tot)}} = \frac{E^{(ZF)}}{E^{(turb)} + E^{(ZF)}}$$



Earth environment

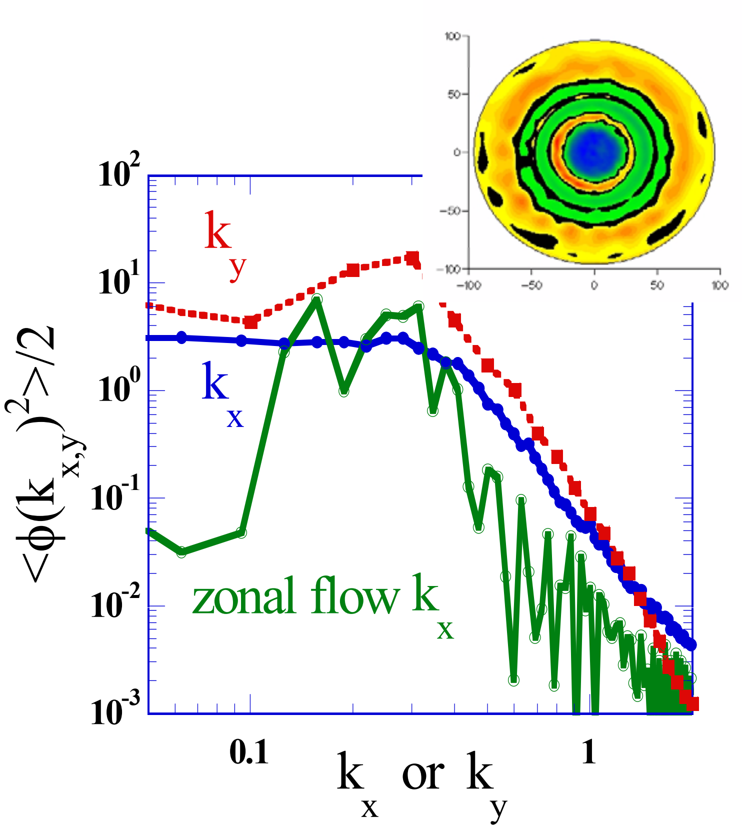


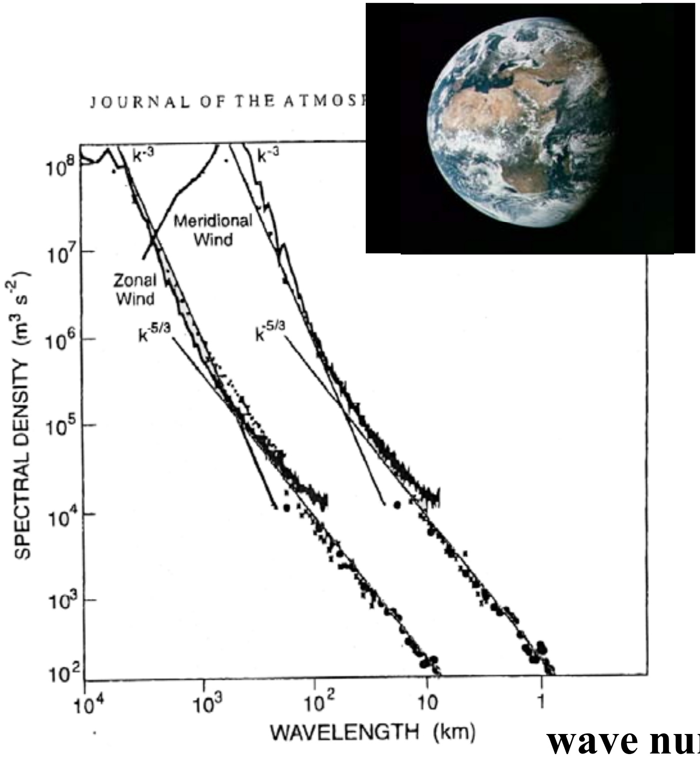
Earth environment ???



Comparison of atmospheric zonal flows

[Koshyk-Hamilton, JAS, 01]





wave number

Hasegawa-Mima Equation

$$\left(1-\nabla^2\right)\frac{\partial \varphi}{\partial t}+v_d\frac{\partial \varphi}{\partial y}-\left[\left(\mathbf{b}\times\nabla\varphi\right)\cdot\nabla\right]\nabla^2\varphi=0$$

Charney Equation

$$\left(1-\nabla^2\right)\frac{\partial h}{\partial t}+v_R\frac{\partial h}{\partial y}-\left[\left(\hat{\mathbf{z}}\times\nabla h\right)\cdot\nabla\right]\nabla^2 h=0$$

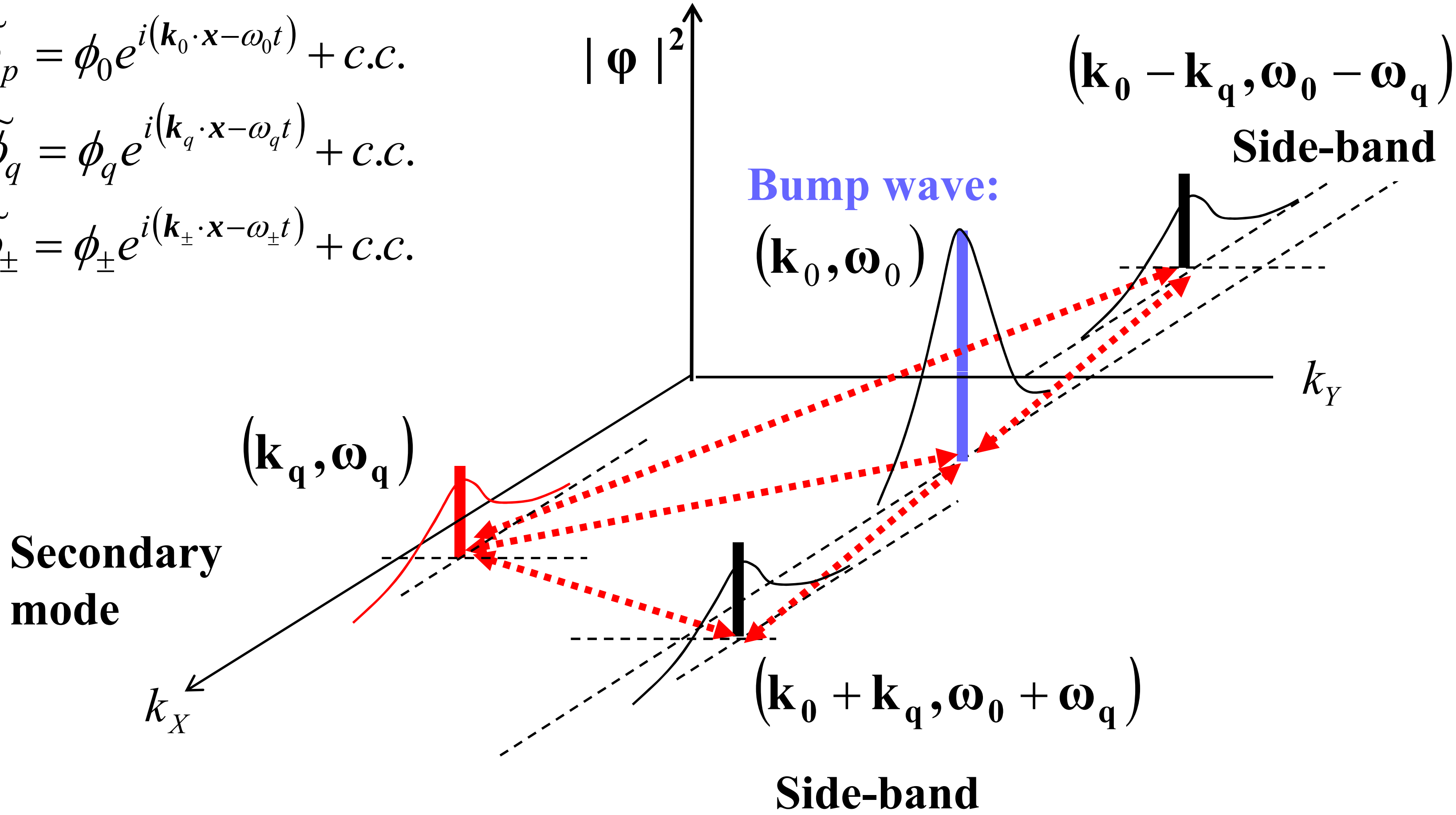
Generation mechanism of secondary fluctuation

Modulational instability : $$\left(1-\nabla_\perp^2\right)\frac{\partial}{\partial \mathbf{t}}\varphi_\mathbf{k}=\frac{\partial}{\partial \mathbf{y}}\varphi_\mathbf{k}+\sum_{\mathbf{k}=\mathbf{k}'+\mathbf{k}''}\left[\varphi_{\mathbf{k}'},\nabla_\perp^2\varphi_{\mathbf{k}''}\right]$$

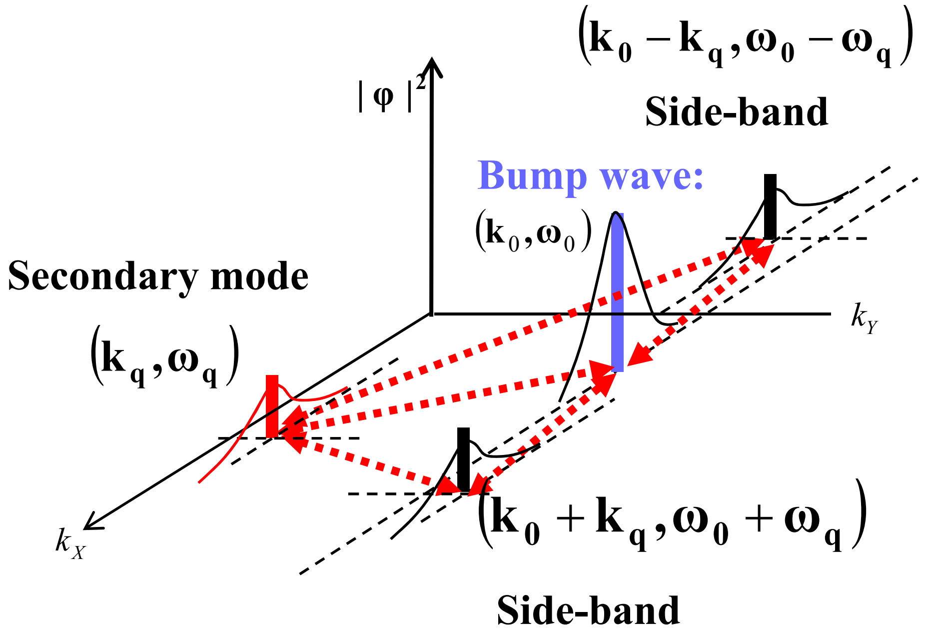
$$\tilde{\phi}_p=\phi_0 e^{i(\boldsymbol{k}_0\cdot\boldsymbol{x}-\omega_0 t)}+c.c.$$

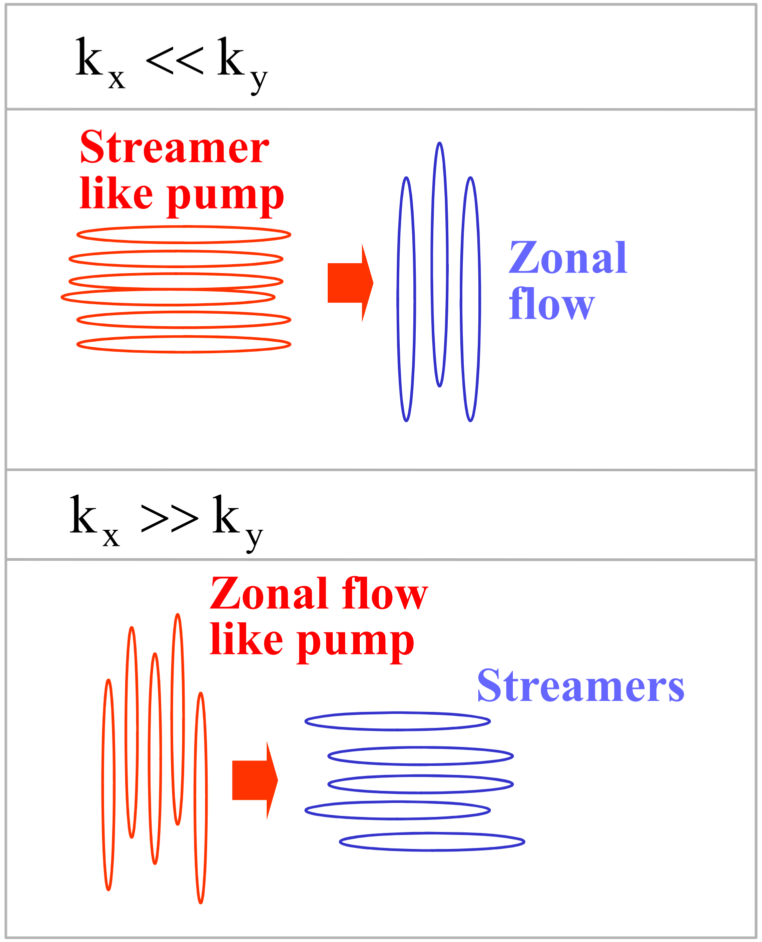
$$\tilde{\phi}_q=\phi_q e^{i(\boldsymbol{k}_q\cdot\boldsymbol{x}-\omega_q t)}+c.c.$$

$$\tilde{\phi}_\pm=\phi_\pm e^{i(\boldsymbol{k}_\pm\cdot\boldsymbol{x}-\omega_\pm t)}+c.c.$$



Control of Secondary Instability





$$\omega_q\left(1+k_q^2\right)+k_{yq} = -\frac{2k_q^2(k_{xq}k_y-k_{yq}k_x)^2(k_0^2-k_q^2)\left\{\omega_q\left[1+k_0^2+k_q^2-4(k_{xq}k_x+k_{yq}k_y)^2/k_q^2\right]+k_{yq}\right\}}{\left[2\omega_0(k_{xq}k_x+k_{yq}k_y)+k_{yq}+\omega_q(1+k_0^2+k_q^2)\right]^2-\left[\omega_0k_q^2+2\omega_q(k_{xq}k_x+k_{yq}k_y)\right]^2}|\varphi_0|^2$$

高性能プラズマの崩壊過程： 突発的な崩壊

線形不安定な初期条件

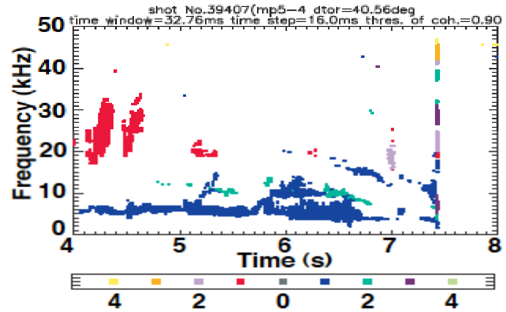
↓

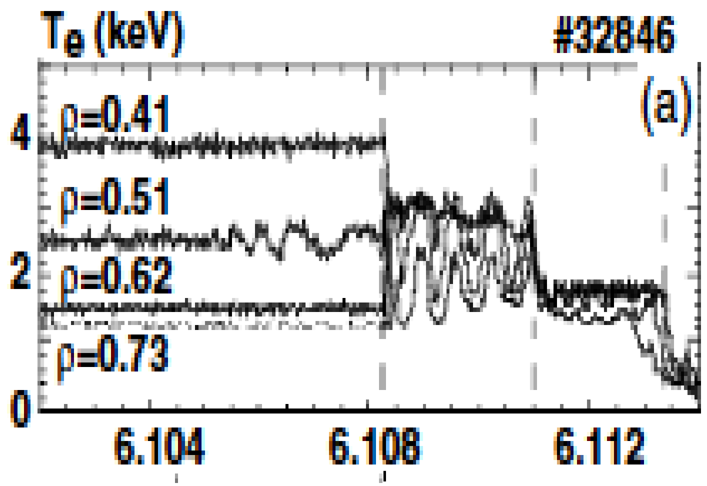
線形不安定性
非線形飽和・崩壊

長時間の準備期間
（初期状態を作る物理過程）

↓ トリガー

非線形不安定性
非線形飽和・崩壊





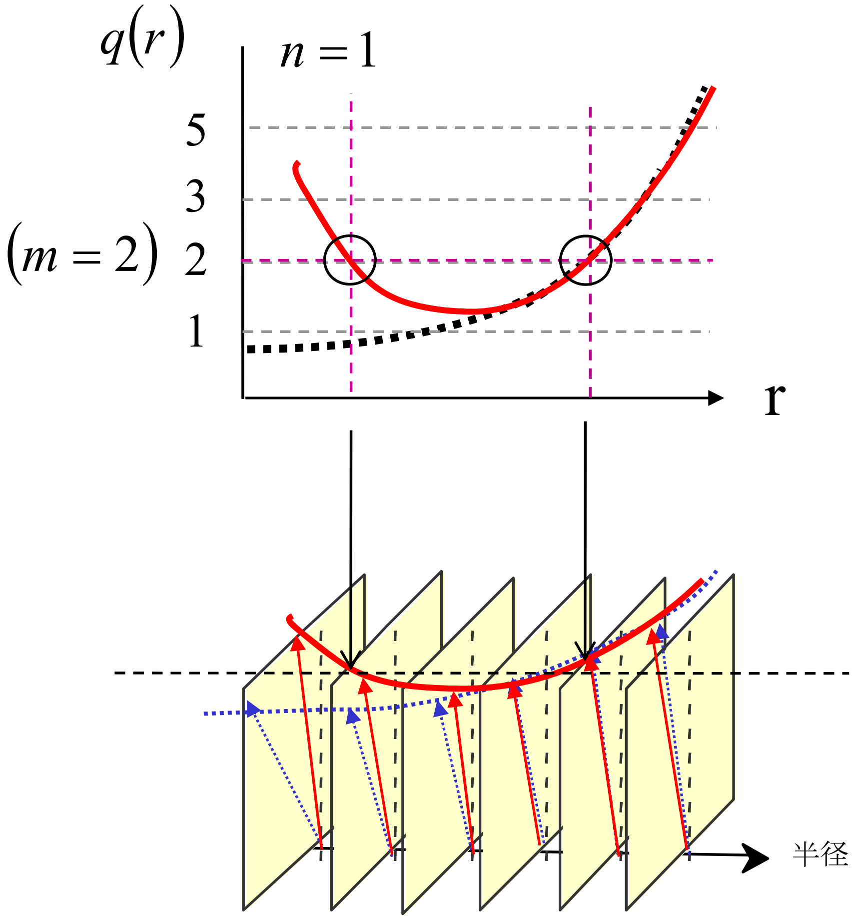
長時間の前兆挙動
（数 msec）

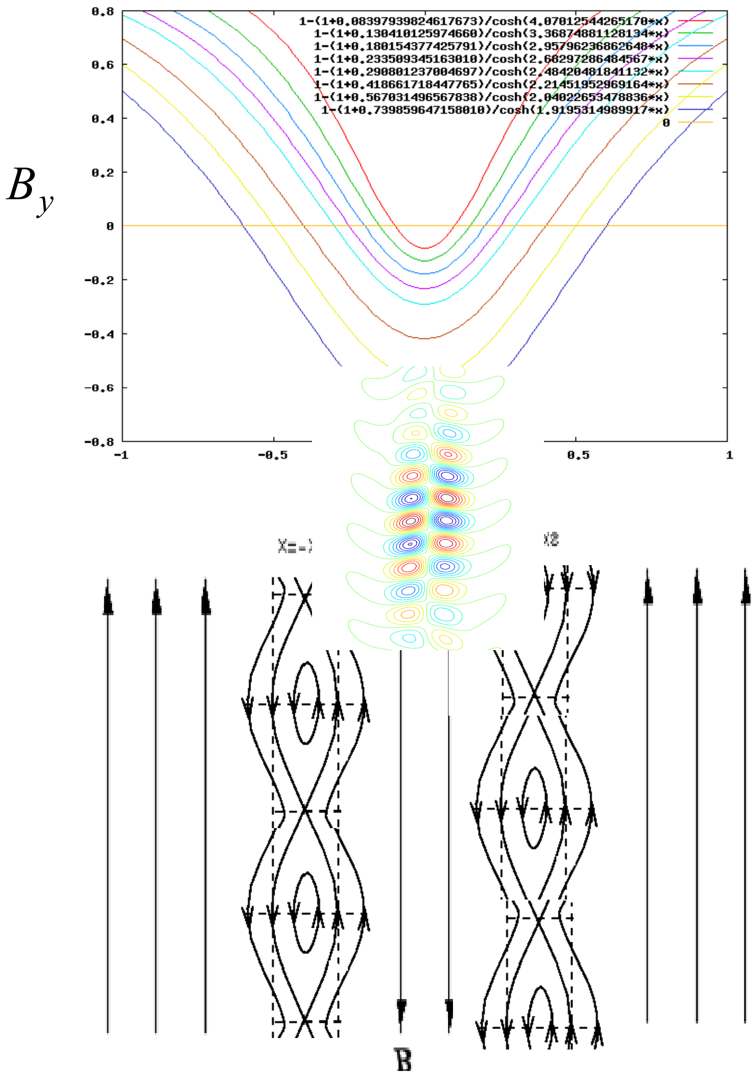
爆発的崩壊
（数十 μsec）

Dynamics of mixed MHD-Turbulence system

by Ms. Janvier Miho

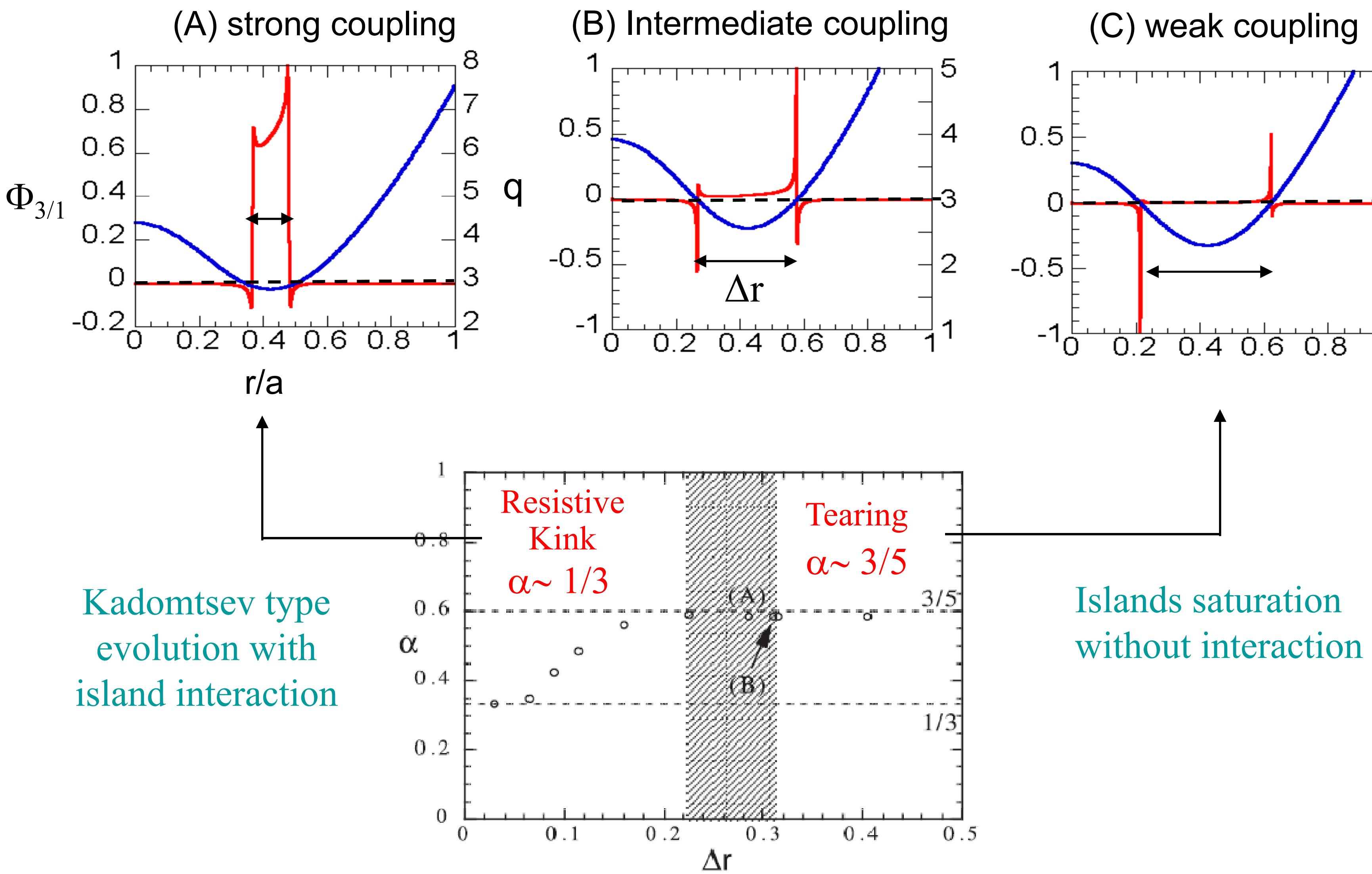
$$q(r) \equiv \frac{r}{R}\frac{B_T}{B_P} \sim \frac{m}{n} \qquad \hat{s} = \frac{r}{q}\frac{\partial q}{\partial r}$$





Linear property of double tearing mode

Resistivity dependence of growth rate $\gamma \sim \eta^{\alpha}$ $\alpha = 1/3 \sim 3/5$

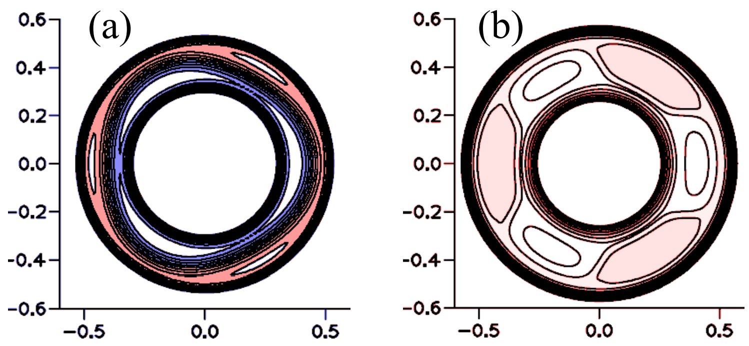


Non-linear destabilization of DTM and the structure (1)

Y. Ishii, M. Azumi and Y. Kishimoto, Phys. Rev. Lett. 89, 205002 (2002)

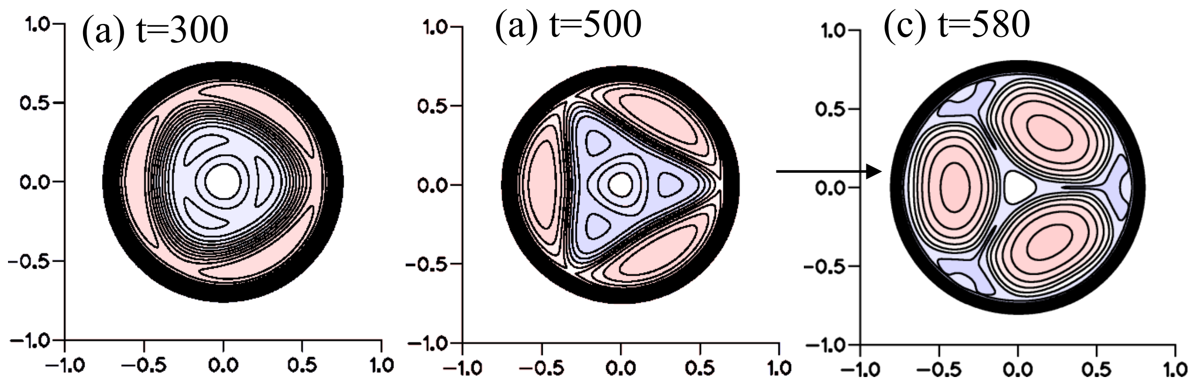
"Structure-driven Nonlinear Instability of DTMs and the abrapt growth after long time scale evolution"

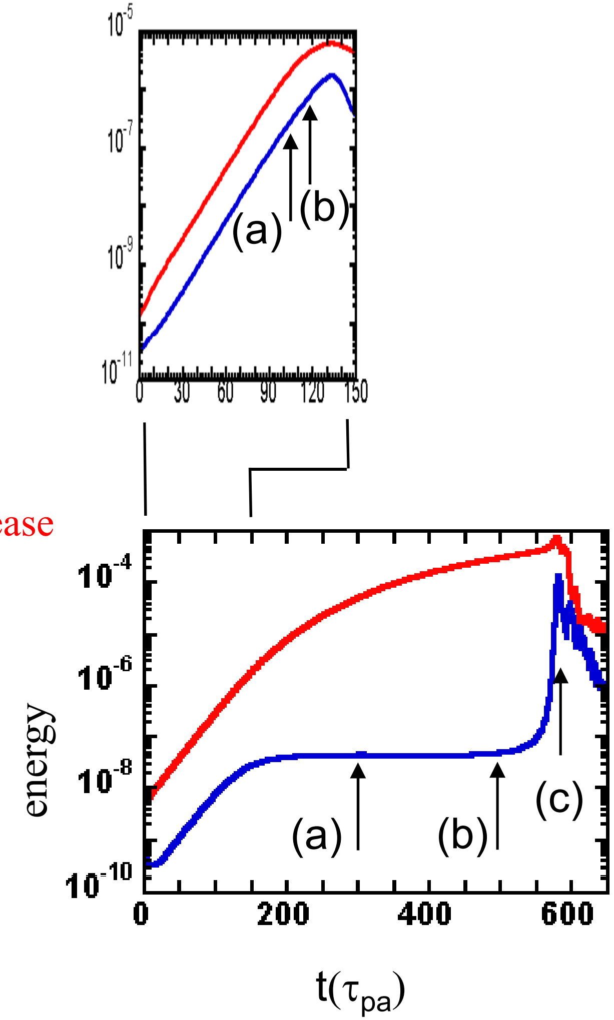
(A) strong coupling



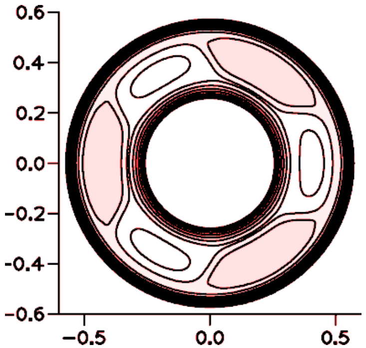
(C) weak coupling

Fast reconnection and explosive energy release

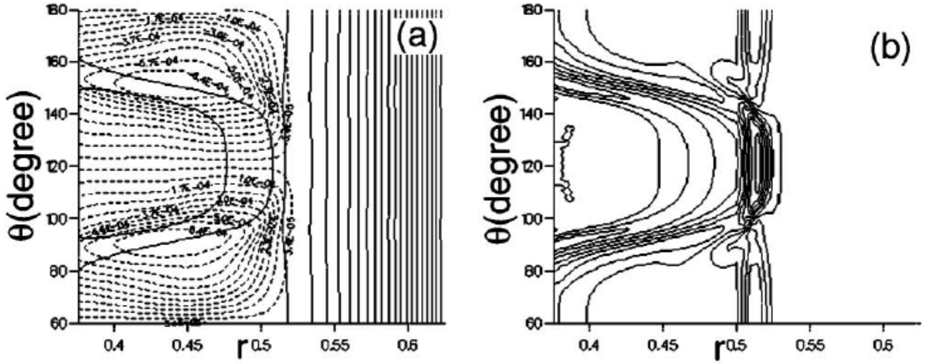




Non-linear de-stabilization of DTM and the structure (2)



(A) strong coupling

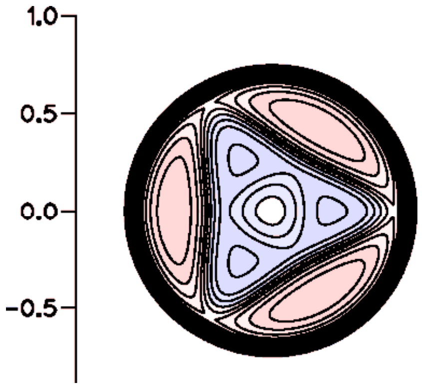


Magnetic flux inflow > Reconnection rate

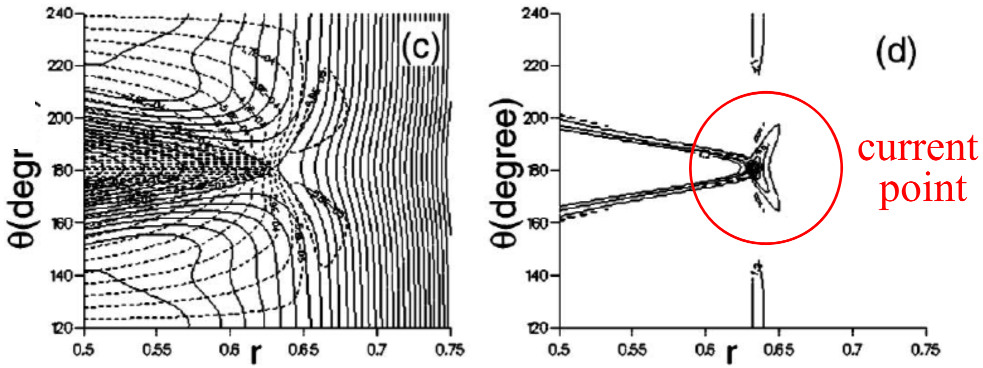
Island evolution is governed
by the global vortex flow

Island evolves with the resistive
time scale for long term

Magnetic flux inflow < Reconnection rate



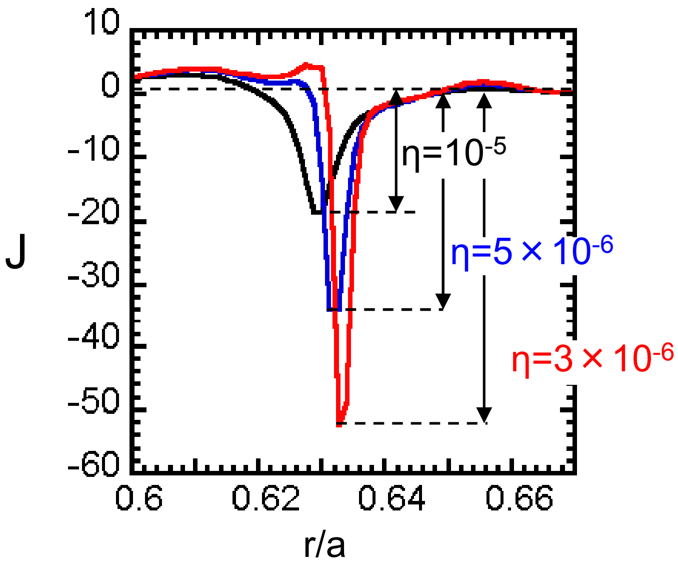
(C) weak coupling



Helical flux and potential

Current
Current

Characteristics of current point

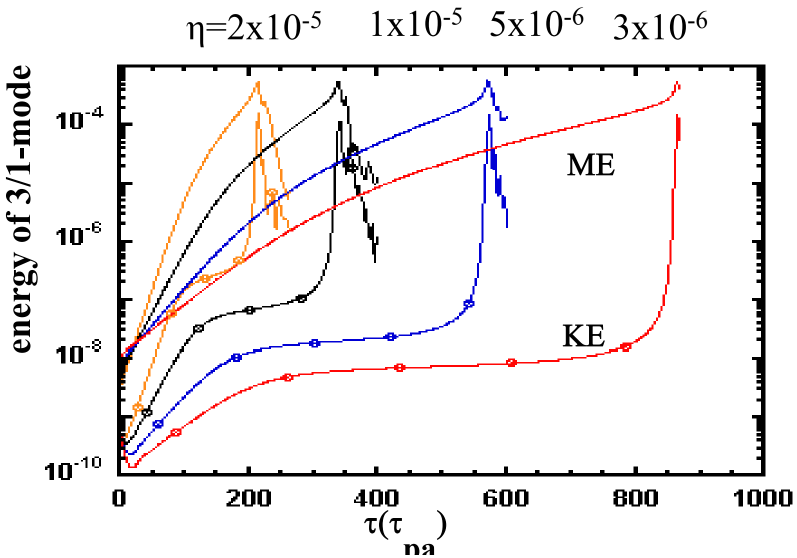


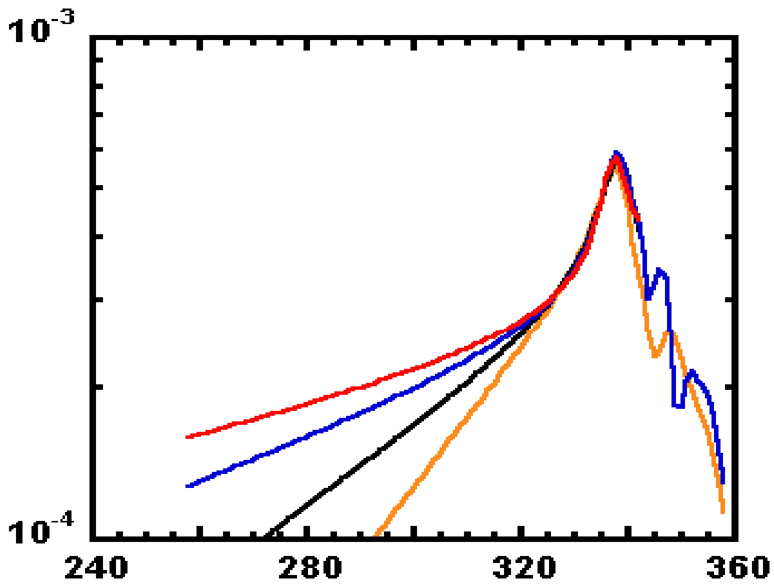
Reconnection rate ηJ is almost constant
w/o depending on η.

Growth rate in the explosive phase
weekly depends on "resistivity"

$$\gamma \sim \eta^{\alpha} \ \ (\alpha \sim 0)$$

Possible trigger mechanism of
"resistivity free" explosive growth ?





Possible mechanism of nonlinear instability (1)

- Linear drive via equilibrium distortion

W. Park, E.D. Fredrickson et al., , Phys. Rev. Lett. 75, 1763 (1995)
"High-b Disruption in Tokamaks"

Helical equilibrium due to low m/n=1/1 kink mode causes
the local pressure steepening, leading to high m/n ballooning mode

Linearly stable for 2D, but nonlinearly unstable for 3D with helical distortion

Explosive distraction due to different scale coupling, leading to thermal quench

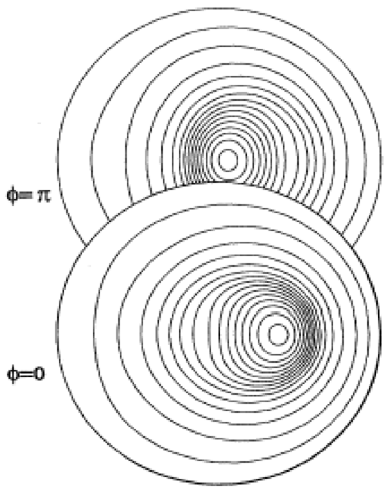


FIG. 2. Pressure contours of the 3D equilibrium.

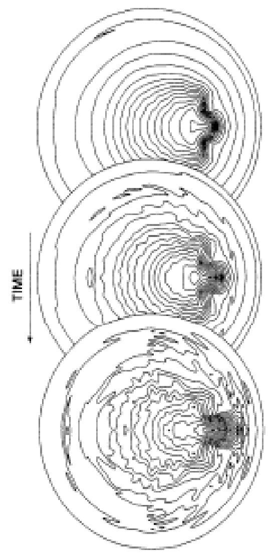


FIG. 4. The nonlinear time development of the pressure.

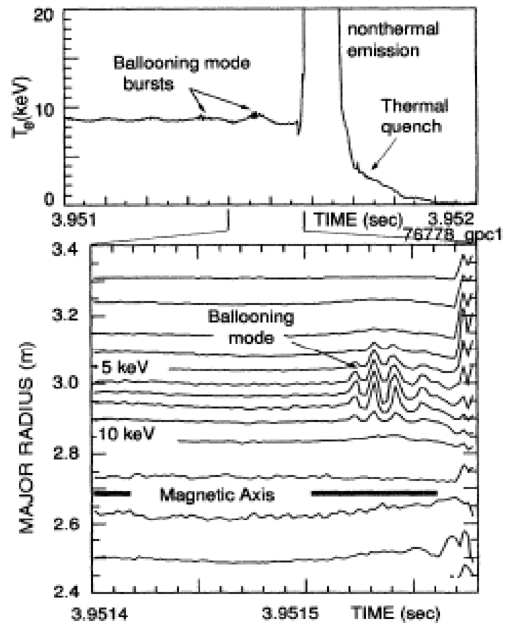


FIG. 7. The experimental ECE signals.

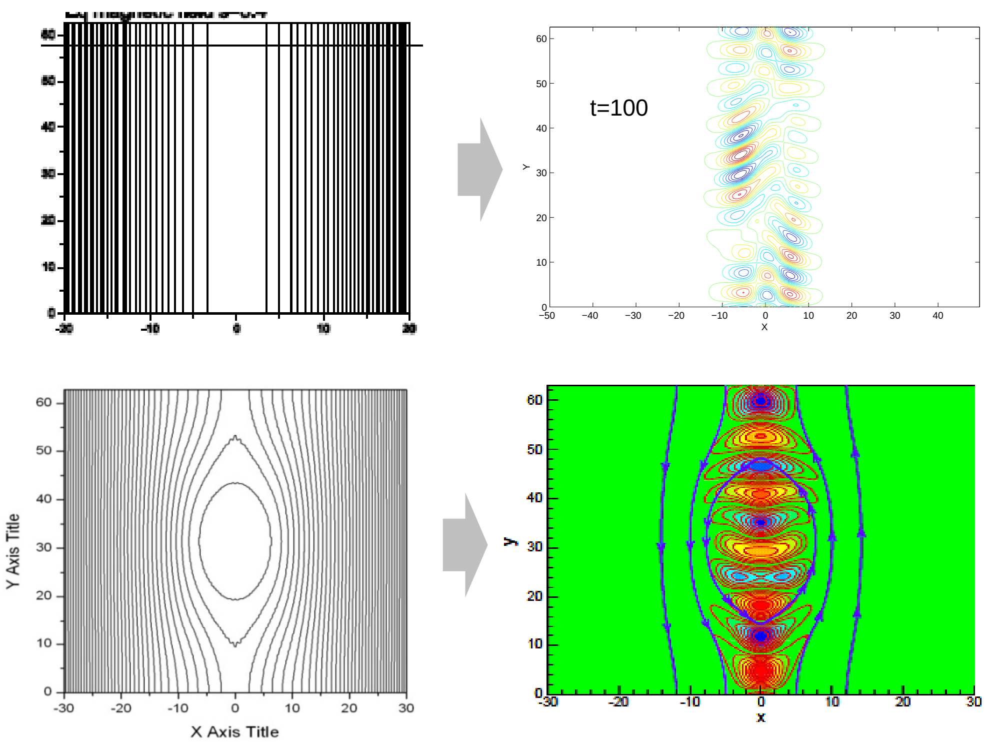
Possible mechanism of nonlinear instability (2)

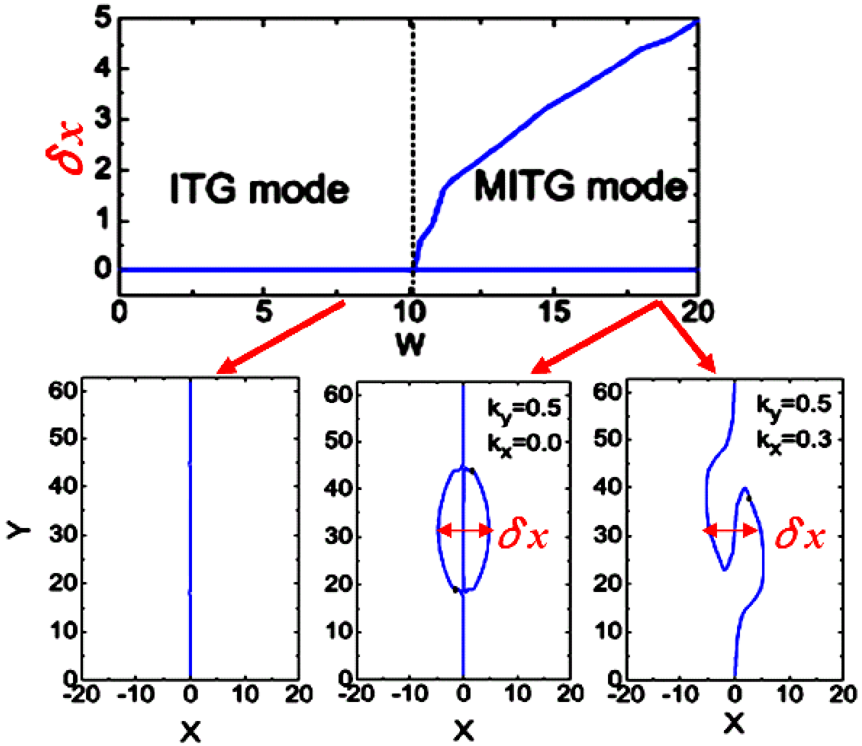
- Linear drive via equilibrium distortion

Slab equilibrium with magnetic island

$$\mathrm{B} = B_T \hat{e}_z + \hat{e}_z \times \nabla \psi \qquad \psi = \hat{s} x^2 / 2 + \tilde{\psi} \cos(k_T y)$$

$$\nabla_{\parallel} f = \hat{z} \times \nabla \psi \cdot \nabla_{\perp} f = \hat{s} x \partial_y f + \left[\tilde{\psi} \cos(k_T y), f \right] \qquad k_{//} = \hat{s} x + \frac{\partial \tilde{\psi}}{\partial x} \cos(k_T y) = 0$$





Multiple resonance surface in space and solar plasmas

Multiple current sheets
i.e., Multiple resonance surfaces

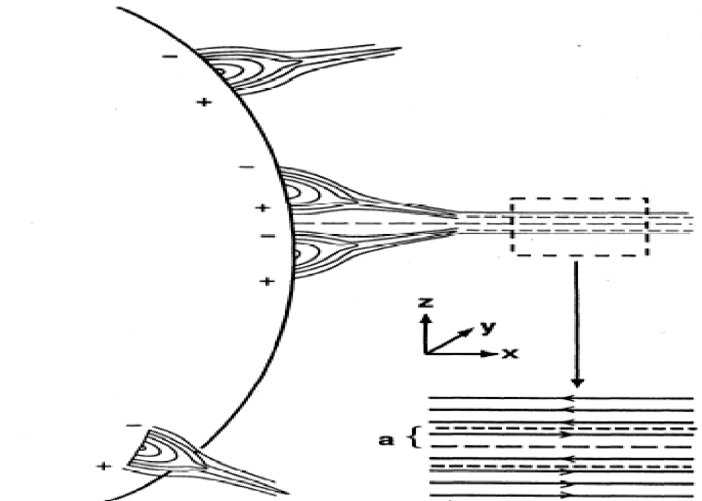
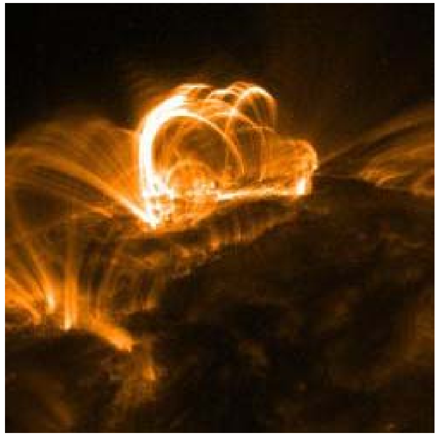
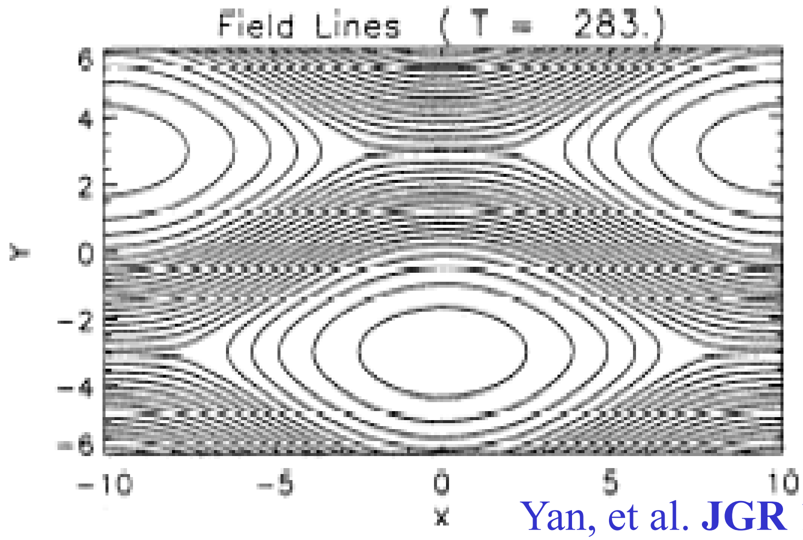


Figure 1. Model of two adjoining coronal helmet streamers and definition sketch of triple current sheet.

R.B.Dahlburg, *et al.* **JGR** 1995



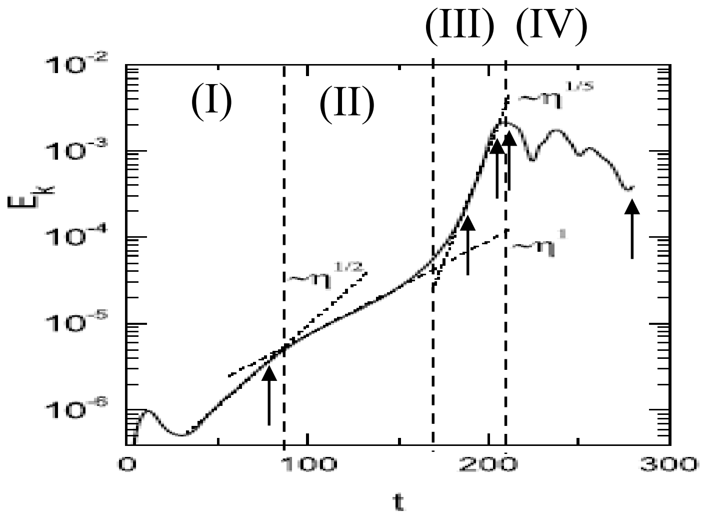


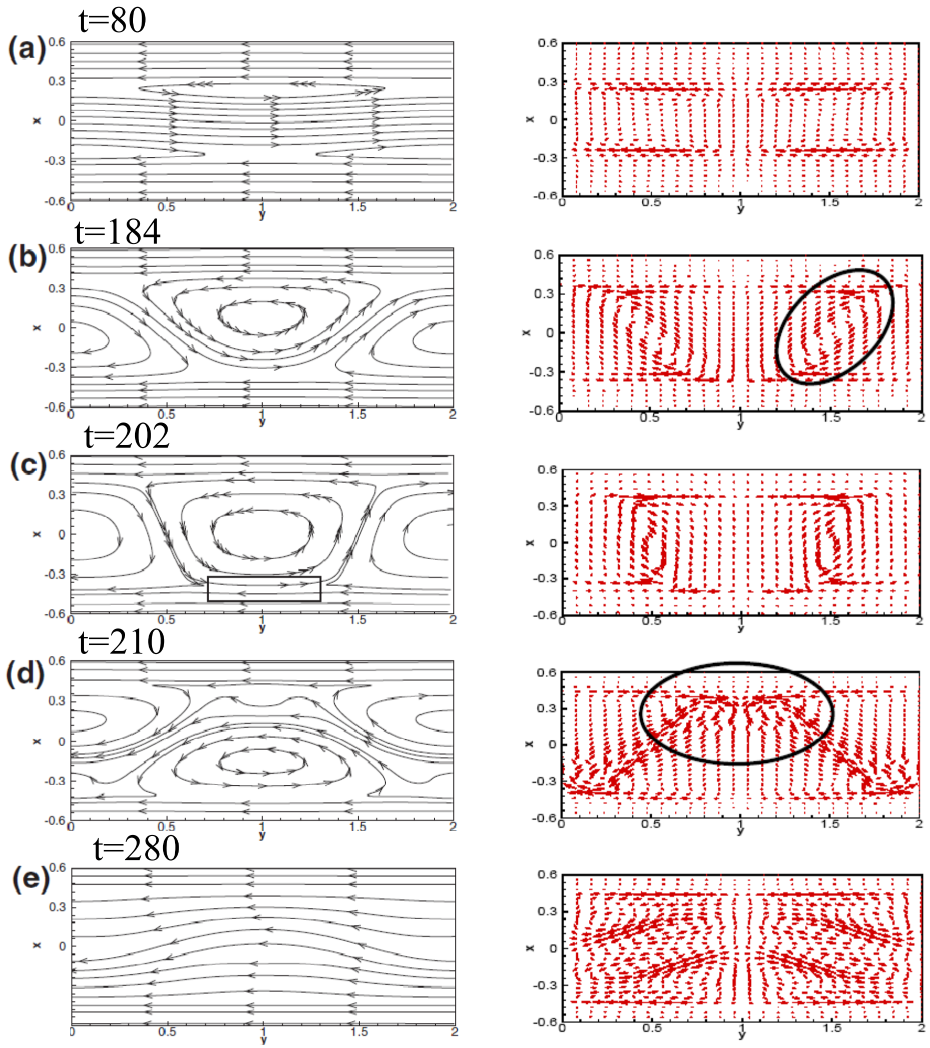
Yan, et al. **JGR** 1994

Neighboring islands drive each other strongly

Fast reconnection regime of DTM in slab geometry

Z.X. Wang, X.G. Wang, J.Q. Dong et al., Phys. Rev. Lett. 99, 185004 (2007)
"Fast Reconnection Regime in the nonlinear evolution of Double Tearing Modes"





- Fast reconnection can take place with no triangular deformation of the magnetic island
- Fast growth is resulting from the neighboring magnetic separatrix merging and equivalent inward flux driven
- Strong vortex shear flow is generated at the boundaries of magnetic islands

Z.X. Wang, X.G. Wang, J.Q. Dong, Y. Kishimoto, and J.Q. Li, Phys Plasmas 15, 082109 (2008)

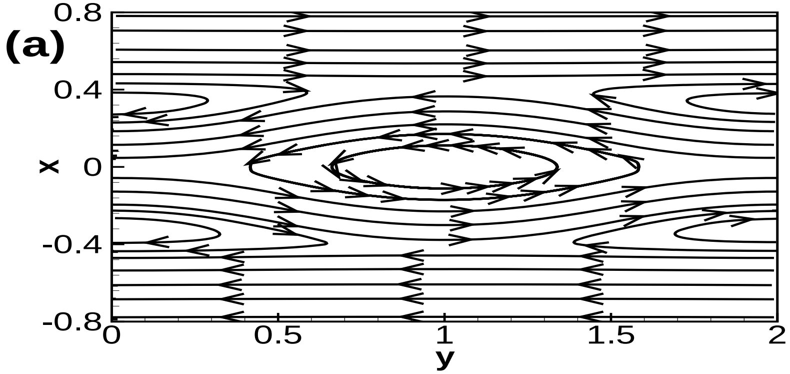
Magnetic configuration

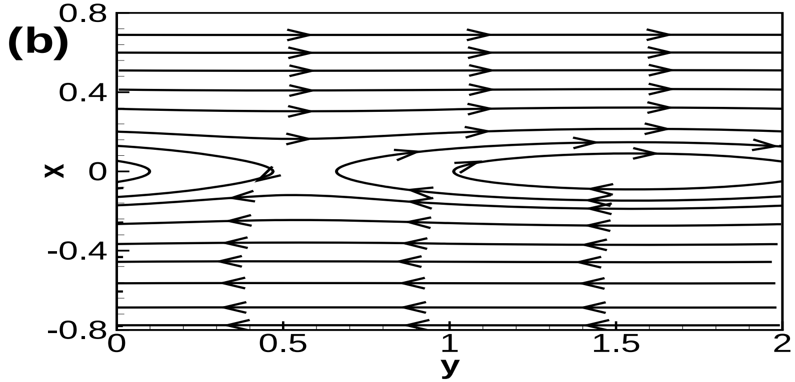
Z.X. Wang, X.G. Wang, J.Q. Dong et al., Phys. Rev. Lett. 99, 185004 (2007)

General rule in predicting the final state of multiple-resonant-reconnection

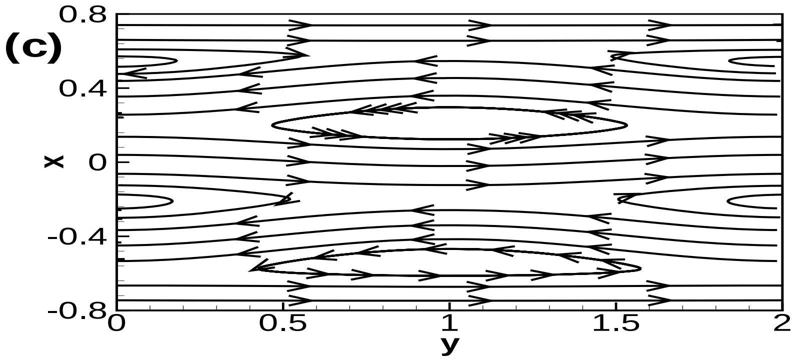
Initial **Saturated**

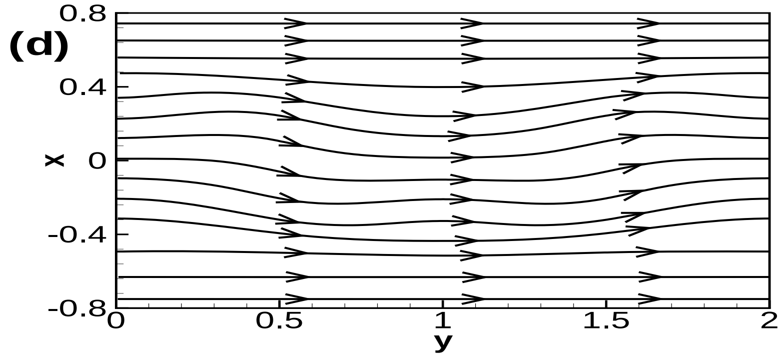
odd resonant surfaces → Single magnetic island

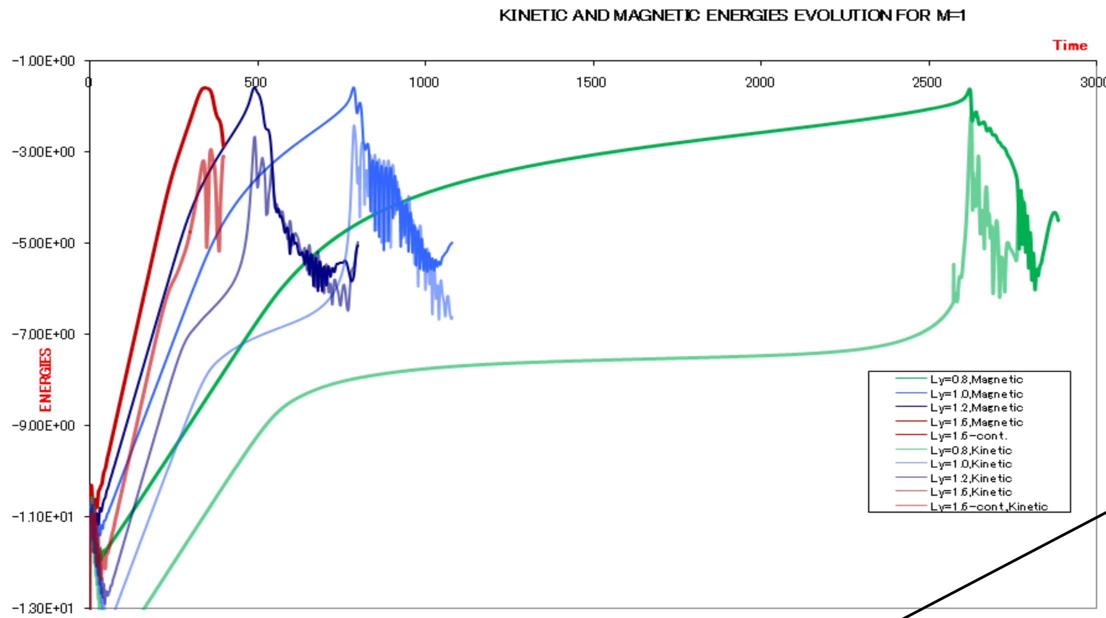




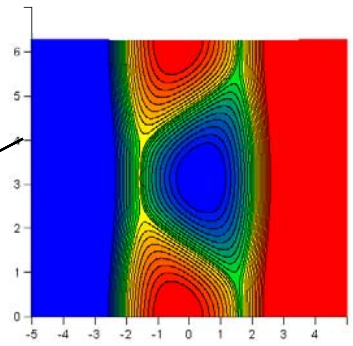
even resonant surfaces → parallel magnetic field lines

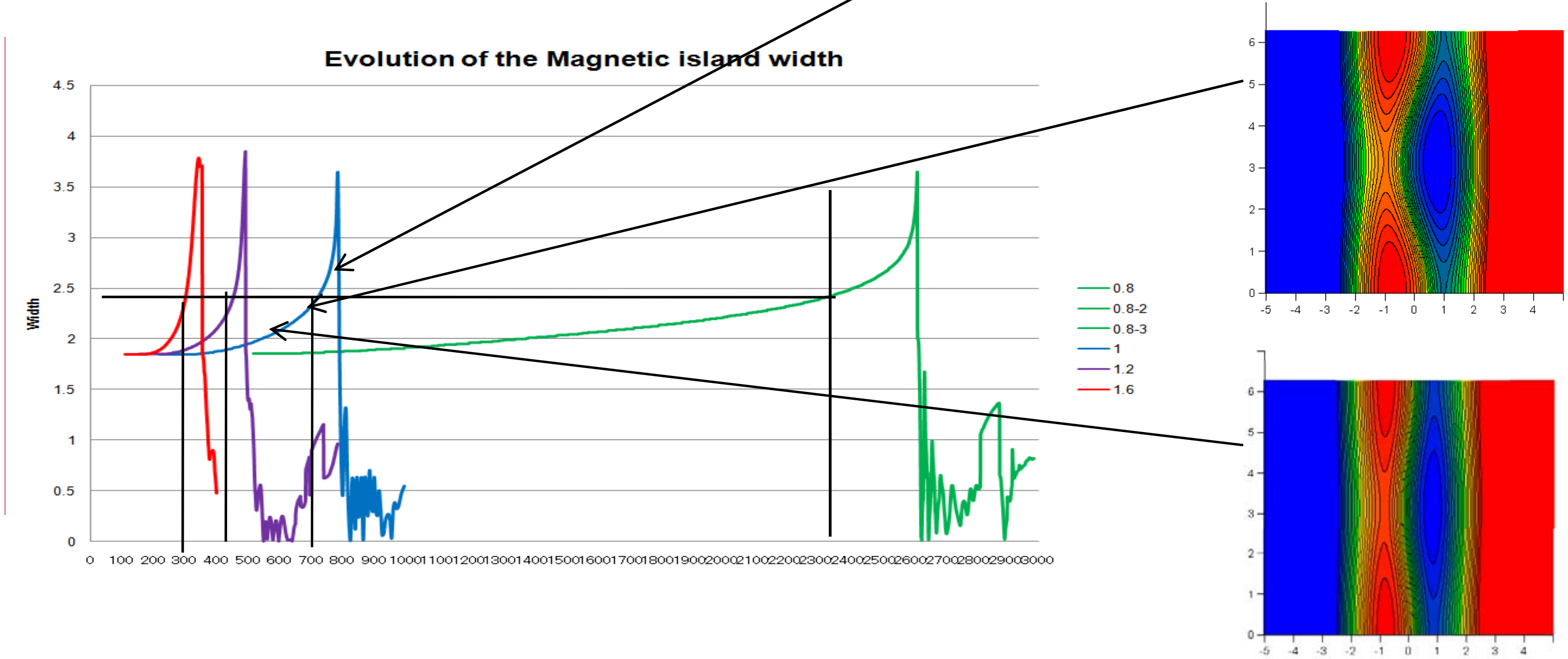


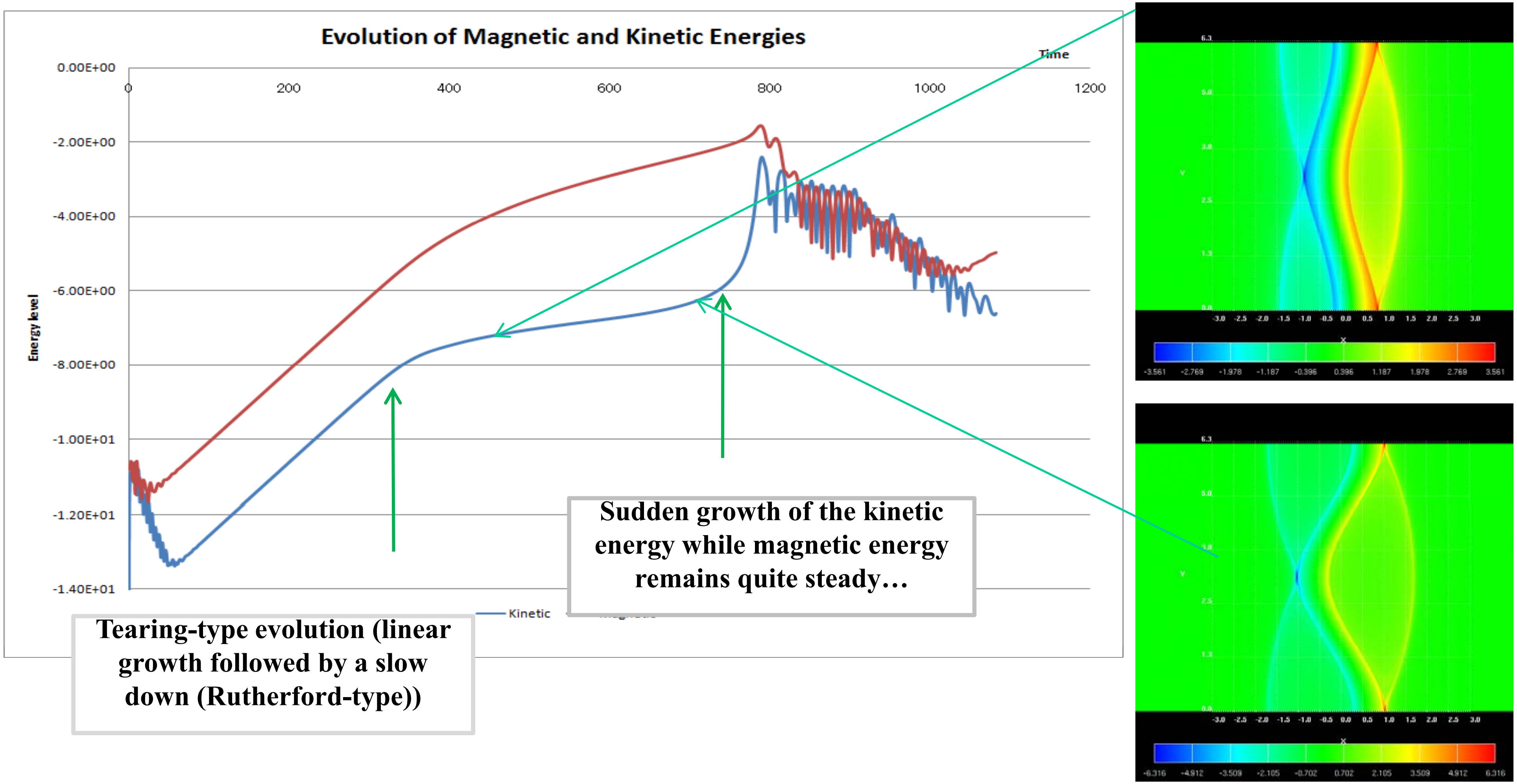


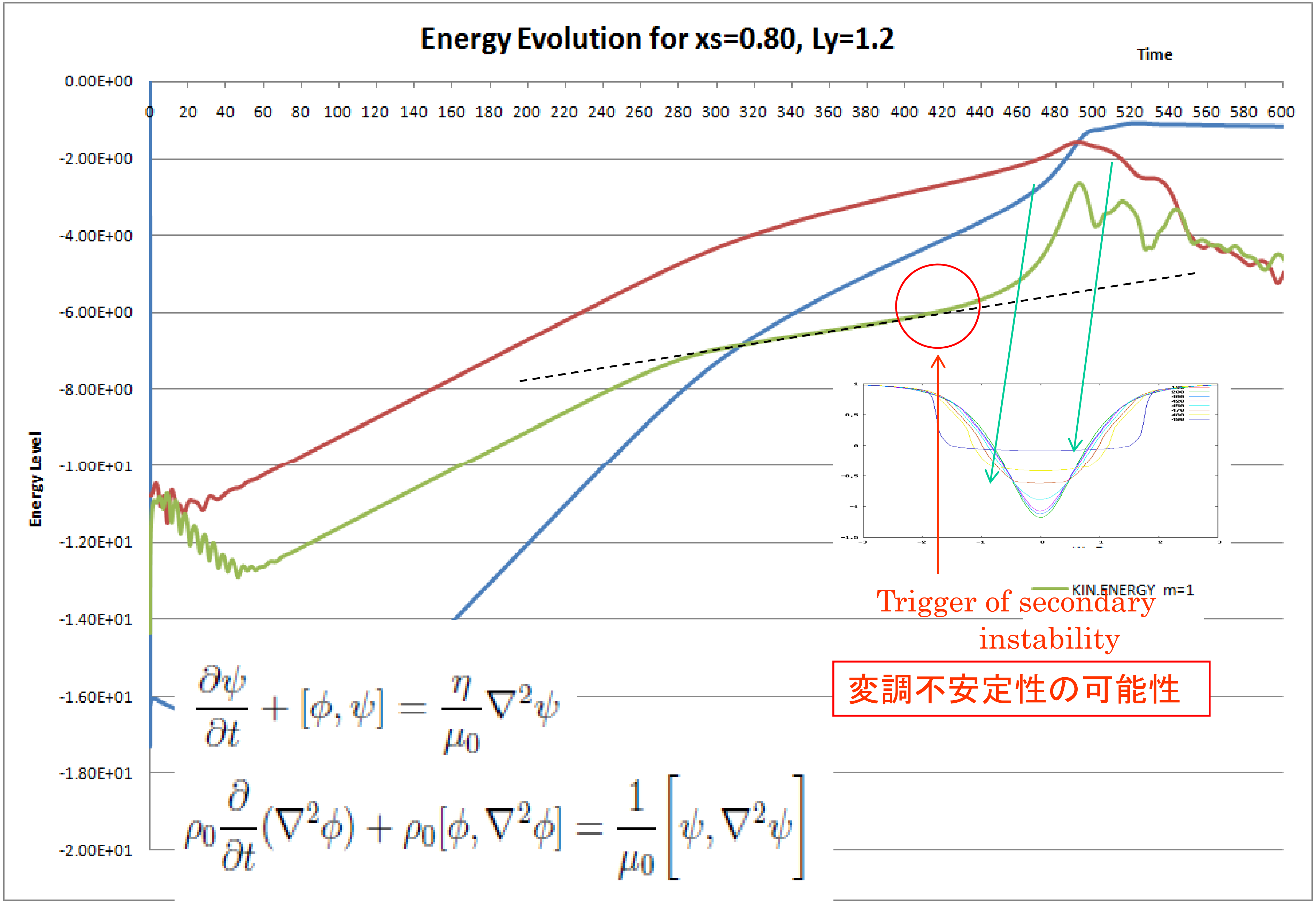


$$B_{oy}(x) = 1 - \frac{(1+B_C)}{\cosh(\zeta x)}$$






Evolution of Magnetic and Kinetic Energies
Time
0
200
400
600
800
1000
1200
0.00E+00
-2.00E+00
-4.00E+00
-6.00E+00
-8.00E+00
-1.00E+01
-1.20E+01
-1.40E+01
Energy level
Kinetic
Tearing-type evolution (linear growth followed by a slow down (Rutherford-type))
Sudden growth of the kinetic energy while magnetic energy remains quite steady…
-3.561
-2.769
-1.978
-1.187
-0.396
0.396
1.187
1.978
2.769
3.561
-6.316
-4.912
-3.509
-2.105
-0.702
0.702
2.105
3.509
4.912
6.316

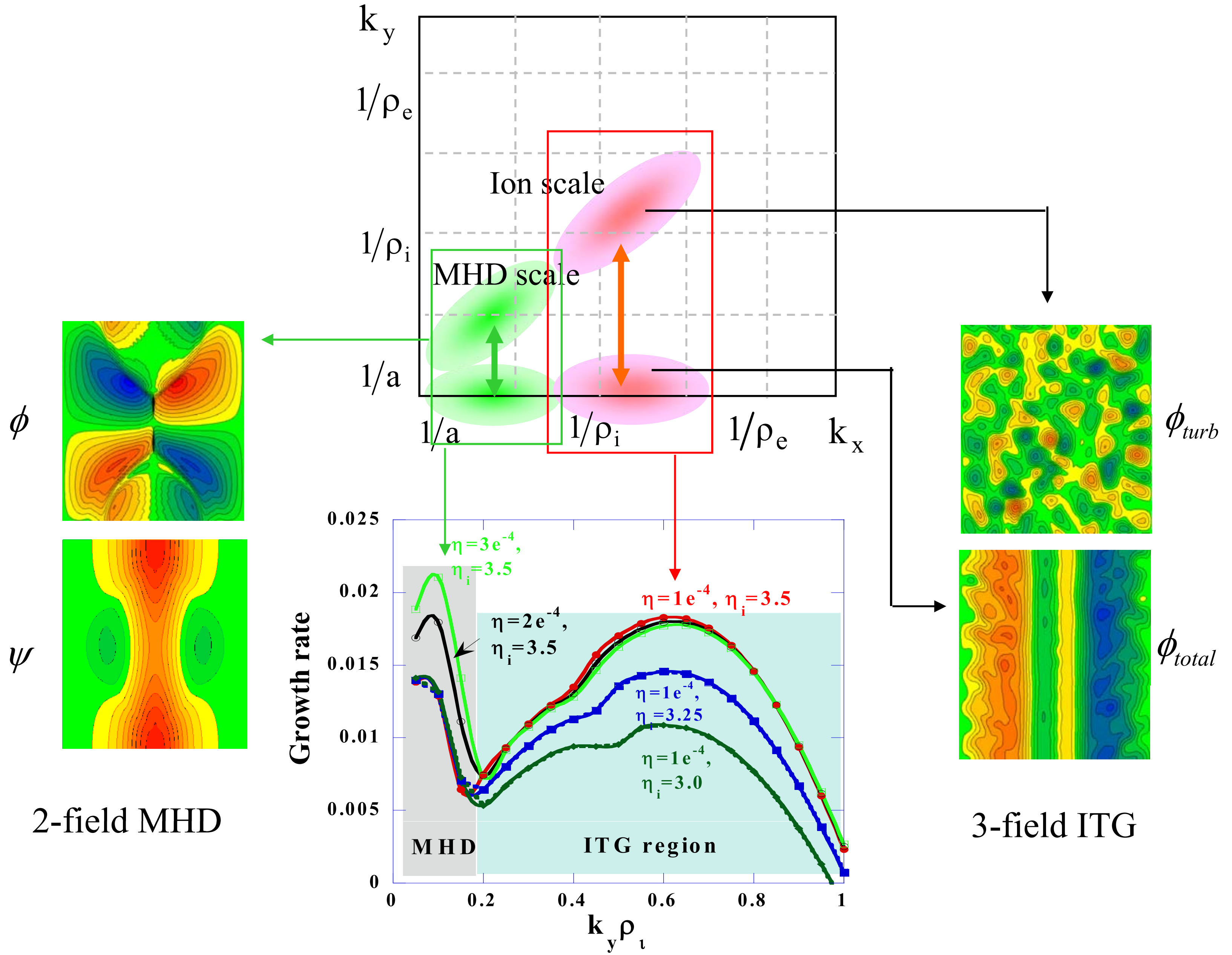


$$\frac{\partial \psi}{\partial t} + [\phi, \psi] = \frac{\eta}{\mu_0} \nabla^2 \psi$$

$$\rho_0 \frac{\partial}{\partial t}(\nabla^2 \phi) + \rho_0 [\phi, \nabla^2 \phi] = \frac{1}{\mu_0} \left[\psi, \nabla^2 \psi \right]$$

変調不安定性の可能性

Physics ingredients in mixed MHD and turbulence model



Gyrofluid modeling equations

Modeling equation – 5-field EM gyrofluid ITG with MHD in slab
(Miyato et al. PoP 04 for toroidal version)

$$
\begin{cases}
\dfrac{\partial}{\partial t}\nabla_\perp^2\phi = -[\phi,\nabla_\perp^2\phi] + (1+\eta_i)\dfrac{\partial}{\partial y}\nabla_\perp^2\phi + \nabla_{//} j_{//} + D_U\nabla_\perp^4\phi \\
\dfrac{\partial}{\partial t} n = -[n,\phi] + \dfrac{\partial\phi}{\partial y} - \nabla_{//}\upsilon_{//i} + \nabla_{//} j_{//} + D_n\nabla_\perp^2 n \\
\beta\dfrac{\partial}{\partial t}A_{//} = -\nabla_{//}\phi + \tau\nabla_{//} n + \beta\tau\dfrac{\partial A_{//}}{\partial y} - \eta j_{//} + \sqrt{\dfrac{\pi}{2}\tau\dfrac{m_e}{m_i}}\left|\nabla_{//}\right|(\upsilon_{//i} - j_{//}) \\
\dfrac{\partial}{\partial t}\upsilon_{//i} = -[\phi,\upsilon_{//i}] - (1+\tau)\nabla_{//} n - \nabla_{//} T_i + \beta(1+\tau+\eta_i)\dfrac{\partial A_{//}}{\partial y} - \eta j_{//} + D_\upsilon\nabla_\perp^2\upsilon_{//i} \\
\dfrac{\partial}{\partial t}T_i = -[\phi,T_i] - \eta_i\dfrac{\partial\phi}{\partial y} - (\gamma-1)\nabla_{//}\upsilon_{//i} - (\gamma-1)\sqrt{\dfrac{8}{\pi}}\left|\nabla_{//}\right| T_i + D_\upsilon\nabla_\perp^2 T_i
\end{cases}
$$

with $j_{//} = -\nabla_\perp^2 A_{//}$ $\quad A_{//} = -\psi$

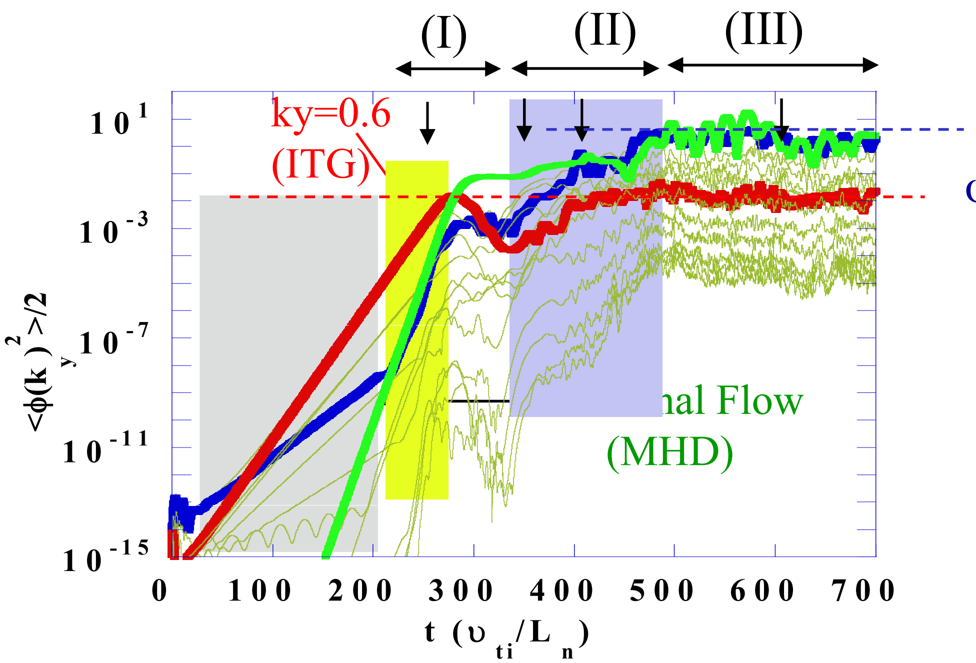
Mean parameters:

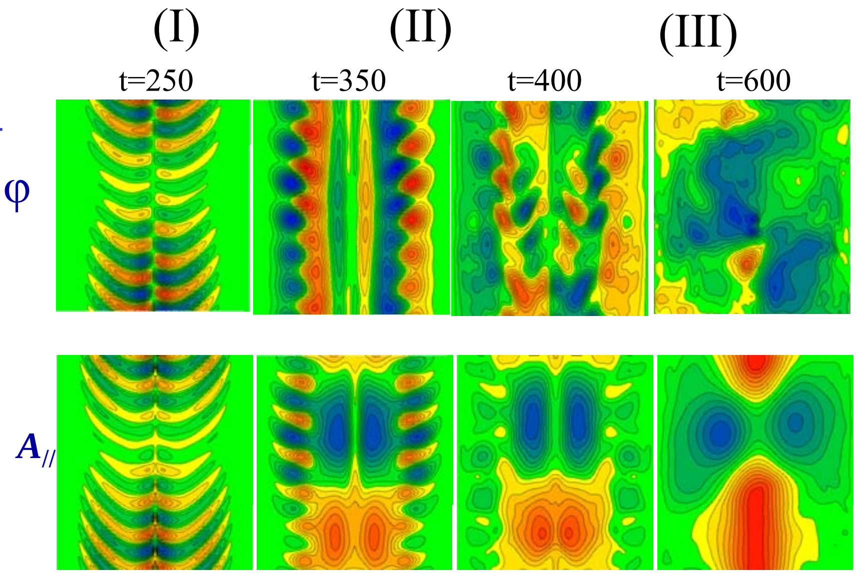
- η_i Ion temperature gradient for ITG fluctuations
- η Resistivity for MHD (kink-tearing) modes

✔ Here the spatio-temporal scales and all corresponding quantities are normalized by ion-scale, similar to those in ITG turbulence.

Evolution of multi-scale turbulence

- Case of Strong ITG drive $\eta_i = 3 \quad \eta = 2\times10^{-4} \quad \beta = 0.01, \quad \hat{s} = 0.2$





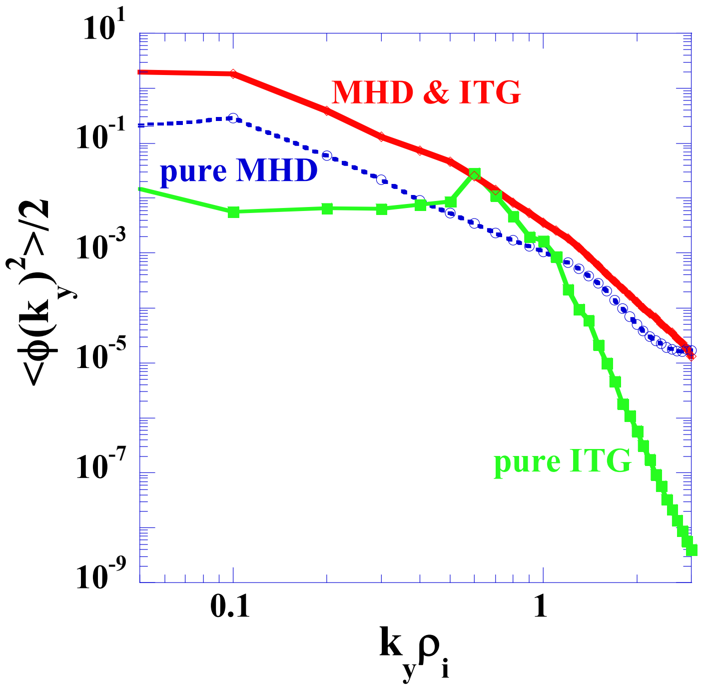
Micro-scale turbulence → Macro-scale MHD mode

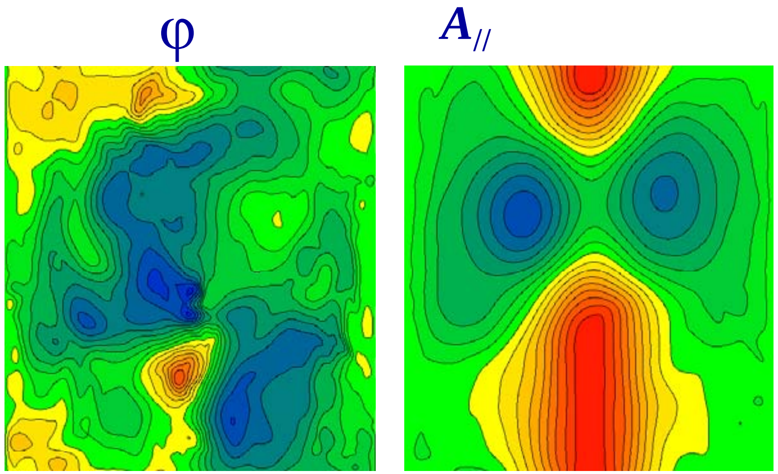
Energy Transfer

Structure and energy in mixed MHD and turbulence

- Spectral dependence on resistivity and ITG

$$\eta = 5 \times 10^{-4} \qquad \eta_i = 2$$





t=600

- "Structure" is supported by MHD mode
- "Energy" is supplied by micro-turbulence

2次的な揺らぎの発生と相互作用

- “1次的な揺らぎ” から “2次的な揺らぎ” の発生

  1） “1次揺らぎ” の平衡変化による“線形”不安定性

  2） “1次揺らぎ” 変調不安定性

  3）　平衡変化　と　変調不安定性 の混在

- 発生した “2次揺らぎ” と“2次揺らぎ”との相互作用

  1） “1次揺らぎ” と“2次・3次揺らぎ” の準定常的な共存
  （自己形成）

  2） 急激（突発的）なダイナミックス
  新しい平衡状態への移行

まとめ

- ▶ 反転磁気シアの磁場配位における揺らぎの構造とダイナミックス
  - “線形的には不安定な系” → 非線形的には有利
（帯状流等の二次的揺らぎ）
  - 大域的な揺らぎ（DTM） → 大域的・爆発的不安定性
電流点形成モデル・スラブモデル

- ▶ マクロなMHDとミクロな乱流（ITG）が共存した系における揺らぎの構造とダイナミックス
  - 異なった揺らぎ間のエネルギー伝達経路
－中間揺らぎと通したモード間結合
－異なったパリティーのモード間の結合（帯状流・帯状磁場）
  - MHDモード → “構造”　ITGモード → “エネルギー”

- ▶ MHDと乱流との相互作用は、線形・非線形的に多彩な構造をダイナミックスを創出